Various

Reciprocal

The reciprocal of a number is 1 ÷ number or the fraction $\frac{1}{\text{number}}$

The reciprocal of 5 is 1 ÷ 5 = 0.2 or the fraction $\frac{1}{5}$

The reciprocal of 0.2 is 1 ÷ 0.2 = 5

The reciprocal of $\frac{2}{5}$ is $1 \div \frac{2}{5} = \frac{5}{2} = 2.5$

The reciprocal of -4 is 1 ÷ -4 = -0.25

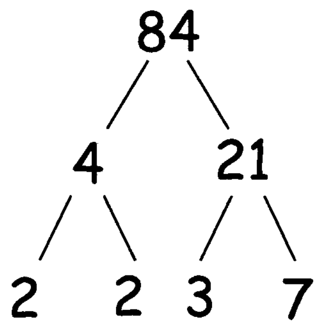
Find the reciprocal of these numbers

1) 10 $1 \div 10 = 0.1$
2) 0.5 $1 \div 0.5 = 2$
3) 2 $1 \div 2 = 0.5$
4) 0.1 $1 \div 0.1 = 10$
5) $\frac{4}{5}$ $1 \div 4/5 = 1 \times 5/4 = 5/4 = 1.25$
6) -5 $1 \div -5 = -1/5 = -0.2$
7) $\frac{1}{3}$ $1 \div 1/3 = 1 \times \frac{3}{1} = 3$
8) -0.125 $1 \div -0.125 = -8$

Prime factor decomposition

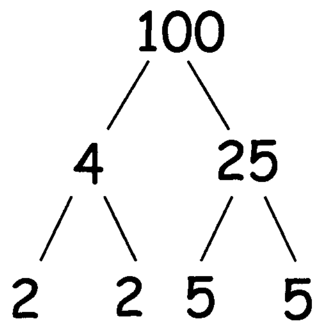
Write the number 84 as a product of prime factors

$$84 = 2^2 \times 3 \times 7$$

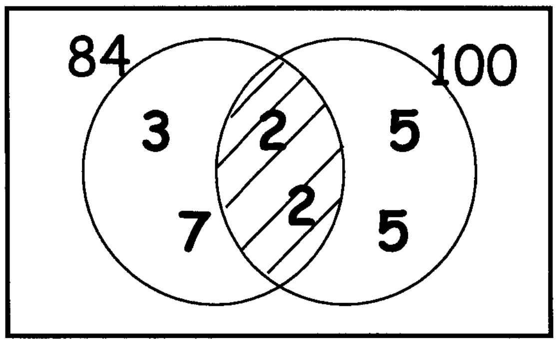


Write the number 100 as a product of prime factors

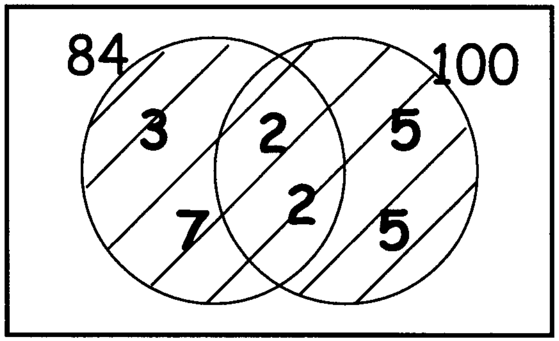
$$100 = 2^2 \times 5^2$$



Find the highest common factor HCF and lowest common multiple LCM of 84 and 100



HCF = 2 X 2 = 4

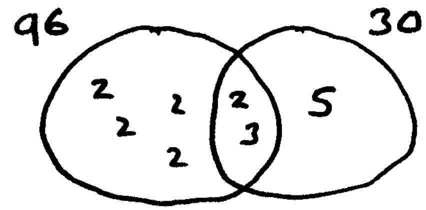


LCM = 3 X 7 X 2 X 2 X 5 X 5 = 2100

Write these pairs of numbers as product of prime factors, then find their HCF and LCM

1) 96 and 30

$96 = 2^5 \times 3$ $\quad 30 = 2 \times 3 \times 5$



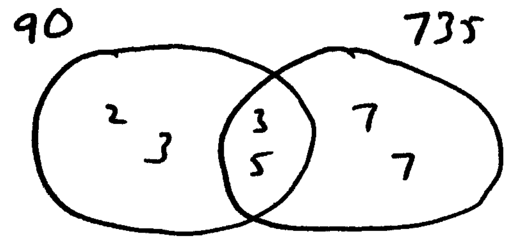
HCF = $2 \times 3 = 6$ $\quad$ HCF = 6

LCM = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 480$

1) 90 and 735

$90 = 2 \times 3^2 \times 5$

$735 = 3 \times 5 \times 7^2$



HCF = $3 \times 5 = 15$

LCM = $2 \times 3 \times 3 \times 5 \times 7 \times 7$

$6 \times 15 \times 49$

$= 4410$

Vocabulary of Algebra

Equation $x + 7 = 10$

Has a specific answer or answers. Eg $x = 3$

Inequality $3x + 7 < 16$

Can be solved but has a range of answers

Formula $A = \pi r^2$

The letters have meaning, A = area, r = radius.
Has a specific purpose.
Generates an answer.

Expression $3x + 7$

Has no equals

Term $3x + 7$

Part of an expression separated by either a plus or minus sign.
The expression $3x + 7$ has an x term and a number term

Identity $x + 2x \equiv 3x$

Just another way of writing the same thing.
Has no answer or an infinite set of answers.
Has its own symbol $\equiv$

Types of Data

Quantative Data - data that can be counted of measured using number. E.g. Age, height, shoe size

Qualitative Data - data that cannot be measured using number. E.g. Colour, type of pet

Continuous Data - data that can be measured and take any value. E.g. height, weight

Discrete Data - data that can only be counted and take certain values. E.g. shoe size, number of cars

Primary Data - data that you collect yourself. New data

Secondary Data - data that someone else has collected

Comparing Terms

If one expression is the same as another it must have the same number of x^2's, x's and numbers.

$7x - 2 \equiv dx + e$ then d = 7 e = −2

$5x^2 - 3x + 2 \equiv ax^2 + bx + c$ then a = 5 b = −3 c = 2

$fx^2 + 5x - 4 \equiv 2x^2 + gx + h$ then f = 2 g = 5 h = −4

Sometimes you may have to multiply out brackets and simplify before you can compare the terms.

$(x + 2)^2 - 3 \equiv ax^2 + bx + c$ then a = 1 b = 4 c = 1

$x^2 + 4x + 4 - 3$

$x^2 + 4x + 1 \equiv ax^2 + bx + c$

$x^2 + 6x + 1 \equiv (x + d)^2 + e$ then d = 3 e = −8

$x^2 + 6x + 1 = x^2 + 2dx + d^2 + e$ $2d = 6$ $d = 3$

$1 = d^2 + e$ $d = 3$ $1 = 3^2 + e$ $1 = 9 + e$ $e = -8$

Find the value of the missing letters in each question

1) $4x - d \equiv ax + 9$ $a = 4$ $d = -9$

2) $4 + 2x^2 - 3x \equiv ax^2 + bx + c$ $a = 2$ $b = -3$ $c = 4$

3) $3(x + 2) + 4(ax + 1) \equiv 11x + b$ $3x + 6 + 4ax + 4 = x(3 + 4a) + 10 \equiv 11x + b$
$3 + 4a = 11$ $a = 2$ $b = 10$

4) $(x - 5)^2 - 42 \equiv ax^2 + bx + c$
$x^2 - 10x + 25 - 42$ $x^2 - 10x - 17$ $a = 1$ $b = -10$ $c = -17$

5) $x^2 + 2x - 15 \equiv (x + d)^2 + e$ $2d = 2$ $d^2 + e = -15$ $1 + e = -15$
$x^2 + 2dx + d^2 + e$ $d = 1$ $1^2 + e = -15$ $e = -16$

6) $5(2x - 1) - 2(ax - b) \equiv 6x + 3$
$10x - 5 - 2ax + 2b = x(10 - 2a) + 2b - 5$ $6 = 10 - 2a$ $a = 2$ $2b - 5 = 3$ $b = 4$

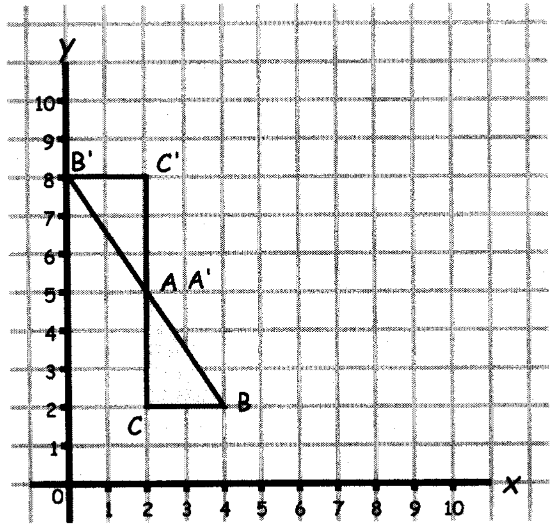
7) $2(x - 3)^2 - 11 \equiv ax^2 + bx + c$
$2(x^2 - 6x + 9) - 11 = 2x^2 - 12x + 18 - 11 = 2x^2 - 12x + 7$ $a = 2$ $b = -12$ $c = 7$

8) $2x^2 + 16x + 5 \equiv a(x + b)^2 + c$
$= a(x^2 + 2bx + b^2) + c$
$= ax^2 + 2abx + ab^2 + c$ (4)

x^2: $a = 2$
x: $16 = 2ab$ $16 = 2 \times 2 \times b$ $b = 4$
Number: $5 = ab^2 + c$ $5 = 32 + c$
$5 = 2 \times 4^2 + c$ $c = -27$

Invariant Points

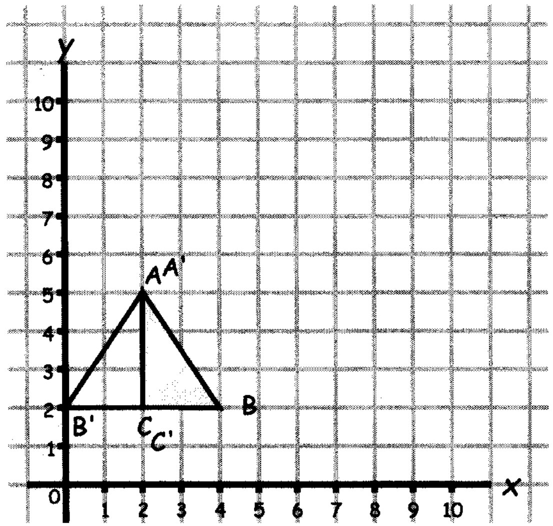
If a point remains in the same position after a transformation it is called Invariant



Carry out the following transformation on the ORIGINAL triangle (shaded)

a) Rotation 180° about (2,5)

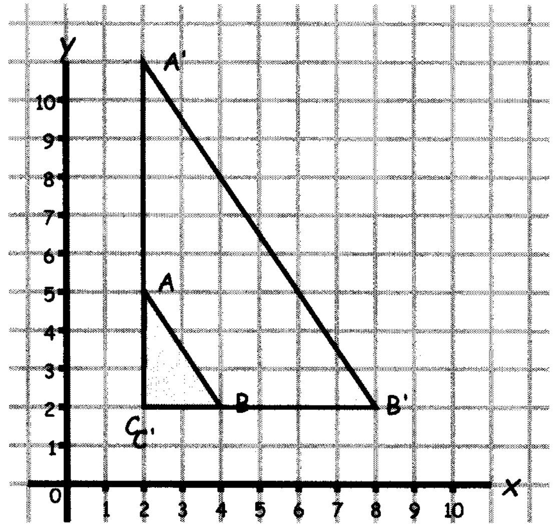
The point A is invariant



Carry out the following transformations on the ORIGINAL triangle (shaded)

b) Reflection in x = 2

The points A and C are invariant, as are all the points on the line AC

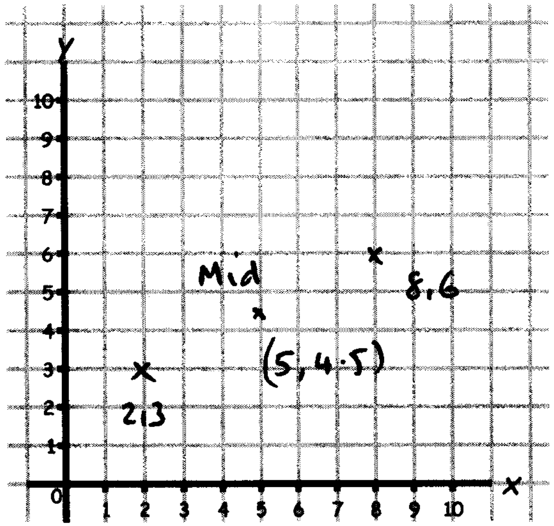


Carry out the following transformations on the ORIGINAL triangle (shaded)

c) Enlargement, scale factor 3, centre (2,2)

The point C is invariant

The coordinates of a the midpoint between 2 points is (add their x coordinates ÷ 2 , add their y coordinates ÷ 2)

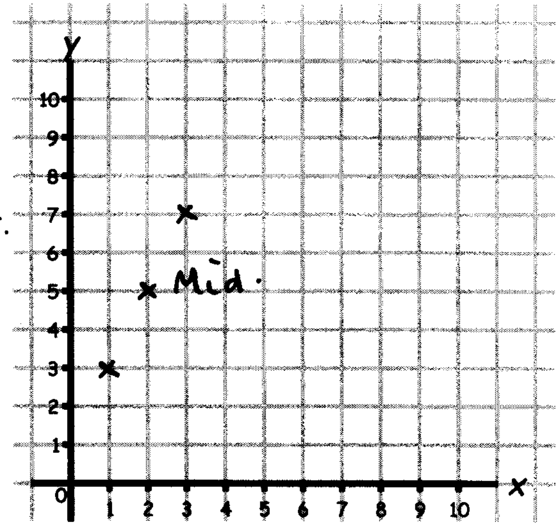


Plot the points (2,3) and (8,6).

Find the coordinates of the midpoint.

$\left(\frac{2+8}{2} , \frac{3+6}{2}\right)$

$(5, 4 \cdot 5)$

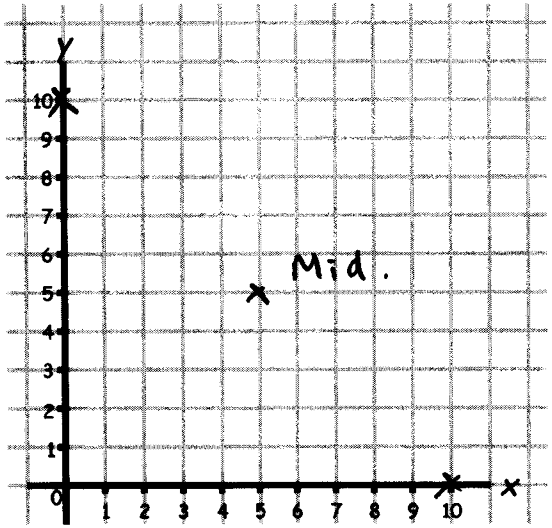


Plot the points (1,3) and (3,7).

Find the coordinates of the midpoint

$\left(\frac{1+3}{2} , \frac{3+7}{2}\right)$

$(2 , 5)$

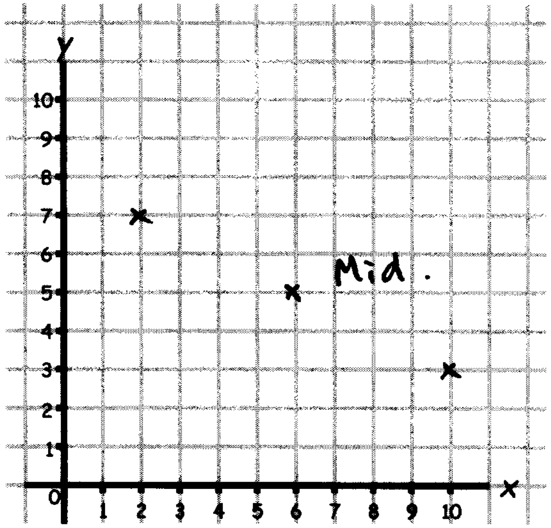


Plot the points (0,10) and (10,0).

Find the coordinates of the midpoint.

$\left(\frac{0+10}{2} , \frac{10+0}{2}\right)$

$(5 , 5)$



Plot the points (2,7) and (10,3).

Find the coordinates of the midpoint

$\left(\frac{2+10}{2} , \frac{7+3}{2}\right)$

$(6 , 5)$

Difference of Two Squares

subtract — two squared terms

Examples

$$a^2 - b^2 = (a - b)(a + b)$$

$$4x^2 - 1 = (2x)^2 - 1^2 = (2x - 1)(2x + 1)$$

$$9x^2 - 25y^2 = (3x)^2 - (5y)^2 = (3x - 5y)(3x + 5y)$$

$2x^2 - 18y^2$ factorise first as neither 2 or 18 are square numbers

$$2x^2 - 18y^2 = 2(x^2 - 9y^2) = 2(x^2 - (3y)^2)$$
$$= 2(x - 3y)(x + 3y)$$

Have a go at these questions

$c^2 - d^2 = (c-d)(c+d)$ or $(c+d)(c-d)$

$x^2 - 9 = (x-3)(x+3)$ or $(x+3)(x-3)$

$16x^2 - 1 = (4x)^2 - 1^2 = (4x-1)(4x+1)$

$4x^2 - 9y^2 = (2x)^2 - (3y)^2 = (2x-3y)(2x+3y)$

$2x^2 - 50y^2 = 2(x^2 - 25y^2)$
$= 2(x^2 - (5y)^2) = 2(x-5y)(x+5y)$

$a^2 - b^2 = (a - b)(a + b)$

$x^2 - 2^2$
$x^2 - 4$ $= (x - 2)(x + 2)$

$(2x)^2 - 3^2$
$4x^2 - 9$ $= (2x + 3)(2x - 3)$

$(3a)^2 - (2b)^2$
$9a^2 - 4b^2$ $= (3a - 2b)(3a + 2b)$