

Quadratics - Solve equations, Quadratic Formula, Complete the square, Graphs

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Quadratics Equations that factorise

1) Solve $x^2 + 7x + 12 = 0$

If the equation does not equal 0, rearrange it so that it does

Factorise the quadratic $(x + 3)(x + 4) = 0$

This represents two numbers multiplied together so that the answer is 0. For example -

If $x = 1$ $(x + 3)(x + 4)$ becomes $4 \times 5 = 20$ and not 0, so $x = 1$ is not an answer

The only possible way that two numbers multiplied together have an answer of 0 is if one of the numbers is 0.

If $(x + 3)(x + 4) = 0$ Then either $x + 3 = 0, x = -3$ OR $x + 4 = 0, x = -4$

There are two answers, $x = -3$ or -4

2) Solve $x^2 - 4x - 12 = 0$

Factorise $(x - 6)(x + 2) = 0$

Either $x - 6 = 0, x = 6$ OR $x + 2 = 0, x = -2$ Answers $x = 6$ or -2

3) Solve $2x^2 = x + 15$ Rearrange to make 0 $2x^2 - x - 15 = 0$

Factorise $(2x + 5)(x - 3) = 0$

Either $2x + 5 = 0, x = -\frac{5}{2}$ OR $x - 3 = 0, x = 3$ Answers $x = -2.5$ or 3

Solve these quadratic equations by factorising

- | | |
|--|--|
| <p>1) $x^2 + 6x + 5 = 0$
 $(x + 5)(x + 1) = 0$
 $x + 5 = 0 \quad x = -5$
 $x + 1 = 0 \quad x = -1$</p> | <p>6) $2x^2 + x - 21 = 0$ $2x + 7 = 0 \quad x = -\frac{7}{2}$
 $(2x + 7)(x - 3) = 0$ $x - 3 = 0 \quad x = 3$</p> |
| <p>2) $x^2 - 8x - 20 = 0$
 $(x - 10)(x + 2) = 0$
 $x - 10 = 0 \quad x = 10$
 $x + 2 = 0 \quad x = -2$</p> | <p>7) $3x^2 - 11x + 6 = 0$ $3x - 2 = 0 \quad x = \frac{2}{3}$
 $(3x - 2)(x - 3) = 0$ $x - 3 = 0 \quad x = 3$</p> |
| <p>3) $x^2 + x - 20 = 0$
 $(x + 5)(x - 4) = 0$
 $x + 5 = 0 \quad x = -5$
 $x - 4 = 0 \quad x = 4$</p> | <p>8) $2x^2 + 7x - 15 = 0$ $2x - 3 = 0 \quad x = \frac{3}{2}$
 $(2x - 3)(x + 5) = 0$ $x + 5 = 0 \quad x = -5$</p> |
| <p>4) $x^2 - 5x + 6 = 0$
 $(x - 2)(x - 3) = 0$
 $x - 2 = 0 \quad x = 2$
 $x - 3 = 0 \quad x = 3$</p> | <p>9) $3x^2 + 13x + 4 = 0$ $x + 4 = 0 \quad x = -4$
 $(3x + 1)(x + 4) = 0$ $3x + 1 = 0 \quad x = -\frac{1}{3}$</p> |
| <p>5) $x^2 - 25 = 0$
 $(x - 5)(x + 5) = 0$
 $x - 5 = 0 \quad x = 5$ $x + 5 = 0 \quad x = -5$ ①</p> | <p>10) $3x^2 - 11x + 10 = 0$ $3x - 5 = 0 \quad x = \frac{5}{3}$
 $(3x - 5)(x - 2) = 0$ $x - 2 = 0 \quad x = 2$</p> |

The quadratic Formula. The equation must equal ZERO. a is the number of x^2 , b is the number of x's and c is the number part.

PUT ANY NEGATIVE NUMBERS IN BRACKETS

Answers to 2dp

$$ax^2 + bx + c = 0$$

$$a = 1$$

$$x^2 + 4x + 2 = 0$$

$$b = 4$$

$$c = 2$$

$$x = \frac{-\boxed{4} \pm \sqrt{\boxed{4}^2 - 4 \times \boxed{1} \times \boxed{2}}}{2 \times \boxed{1}}$$

$$x = -0.59$$

or

$$x = -3.41$$

$$ax^2 + bx + c = 0$$

$$a = 2$$

$$2x^2 + 5x - 1 = 0$$

$$b = 5$$

$$c = -1$$

$$x = \frac{-\boxed{5} \pm \sqrt{\boxed{5}^2 - 4 \times \boxed{2} \times \boxed{-1}}}{2 \times \boxed{2}}$$

$$x = 0.19$$

or

$$x = -2.69$$

$$ax^2 + bx + c = 0$$

$$a = 4$$

$$4x^2 - 3x - 2 = 0$$

$$b = -3$$

$$c = -2$$

$$x = \frac{-\boxed{-3} \pm \sqrt{\boxed{-3}^2 - 4 \times \boxed{4} \times \boxed{-2}}}{2 \times \boxed{4}}$$

$$x = 1.18$$

or

$$x = -0.43$$

$$2x^2 - 5x = 1$$

$$a = 2$$

rearrange so that it equals zero

$$b = -5$$

$$2x^2 - 5x - 1 = 0$$

$$c = -1$$

$$x = \frac{-\boxed{-5} \pm \sqrt{\boxed{-5}^2 - 4 \times \boxed{2} \times \boxed{-1}}}{2 \times \boxed{2}}$$

$$x = 2.69$$

or

$$x = -0.19$$

②

To solve the quadratic equation $ax^2 + bx + c = 0$ use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

NOTE - the equation you are solving must equal 0 before you read off the values of a, b and c.

	Is the equation in the form $ax^2 + bx + c = 0$? If not rearrange it	taken from the equation			Use the equation to get the two answers. Be careful when substituting negative numbers. Round answers to 2 dp
		a number of x^2	b number of x	c the number	
$x^2 + 5x + 6 = 0$	Yes	1	5	6	$x = -2$ or -3
$x^2 + 7x + 5 = 0$	Yes	1	7	5	$x = -0.81$ or -6.19
$5x^2 + 2x = 4$	No $5x^2 + 2x - 4 = 0$	5	2	-4	$x = 0.72$ or -1.12
$3x^2 = 14x - 5$	No $3x^2 - 14x + 5 = 0$	3	-14	5	$x = 4.28$ or 0.39
$x(9x - 1) = 4$	No $9x^2 - x = 4$ $9x^2 - x - 4 = 0$	9	-1	-4	$x = 0.72$ or -0.61
$7 - 2x^2 = 14x$	No $0 = 2x^2 + 14x - 7$ OR $2x^2 + 14x - 7 = 0$	2	14	-7	$x = 0.47$ or -7.47

(3)

Completing the square for $1x^2$ only

$x^2 + 4x + 7$ is to be written in the form $(x + a)^2 + b$

$$x^2 + 4x + 7$$

half this number (the number of x 's) this gives this number

Now consider what would happen if you expanded

$$(x + 2)^2 = x^2 + 4x + 4$$

We want $x^2 + 4x$ but don't want the $+4$, so we subtract this

$$(x + 2)^2 - 4 = x^2 + 4x$$

Put this back in the original expression in place of the $x^2 + 4x$

$x^2 + 4x + 7$ becomes

$$(x + 2)^2 - 4 + 7$$

$$(x + 2)^2 + 3$$

So $x^2 + 4x + 7$ is the same as $(x + 2)^2 + 3$

$a = 2$ and $b = 3$

Complete the square on these quadratic expressions

$$\begin{aligned} 1) x^2 + 2x + 9 &= (x+1)^2 - 1 + 9 \\ &= (x+1)^2 + 8 \end{aligned}$$

$$\begin{aligned} 2) x^2 - 2x + 7 &= (x-1)^2 - 1 + 7 \\ &= (x-1)^2 + 6 \end{aligned}$$

$$\begin{aligned} 3) x^2 - 4x + 10 &= (x-2)^2 - 4 + 10 \\ &= (x-2)^2 + 6 \end{aligned}$$

$$\begin{aligned} 4) x^2 + 6x + 10 &= (x+3)^2 - 9 + 10 \\ &= (x+3)^2 + 1 \end{aligned}$$

$$\begin{aligned} 5) x^2 - 6x - 13 &= (x-3)^2 - 9 - 13 \\ &= (x-3)^2 - 22 \end{aligned}$$

Sketch the graph

$$y = x^2 + 2x - 8$$

1) Find where the graph crosses the y axis, this is when $x = 0$

$$y = 0^2 + 2 \times 0 - 8 = -8$$

2) Find where the graph crosses the x axis, this is when $y = 0$

$$0 = x^2 + 2x - 8$$

Factorise and solve

$$(x + 4)(x - 2) = 0 \quad x = -4 \text{ or } 2$$

3) Find the turning point of the graph by completing the square

$$y = x^2 + 2x - 8 = (x + 1)^2 - 1 - 8$$

$$y = (x + 1)^2 - 9$$

The minimum value a number squared can be is 0. $0^2 = 0$

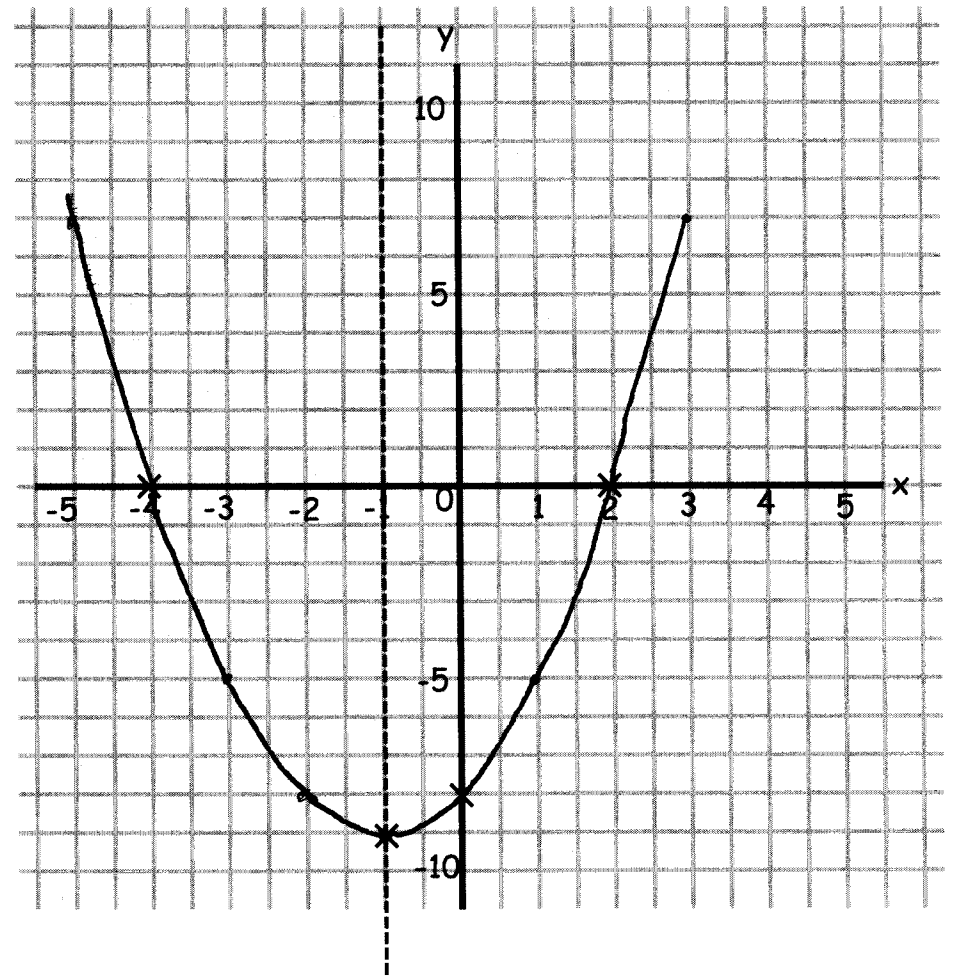
The lowest value y can be is when the squared part is 0.

$$y = 0 - 9 = -9$$

$$\text{so } (x + 1)^2 = 0, \quad x + 1 = 0, \quad x = -1$$

The turning point of the graph is at $(-1, -9)$

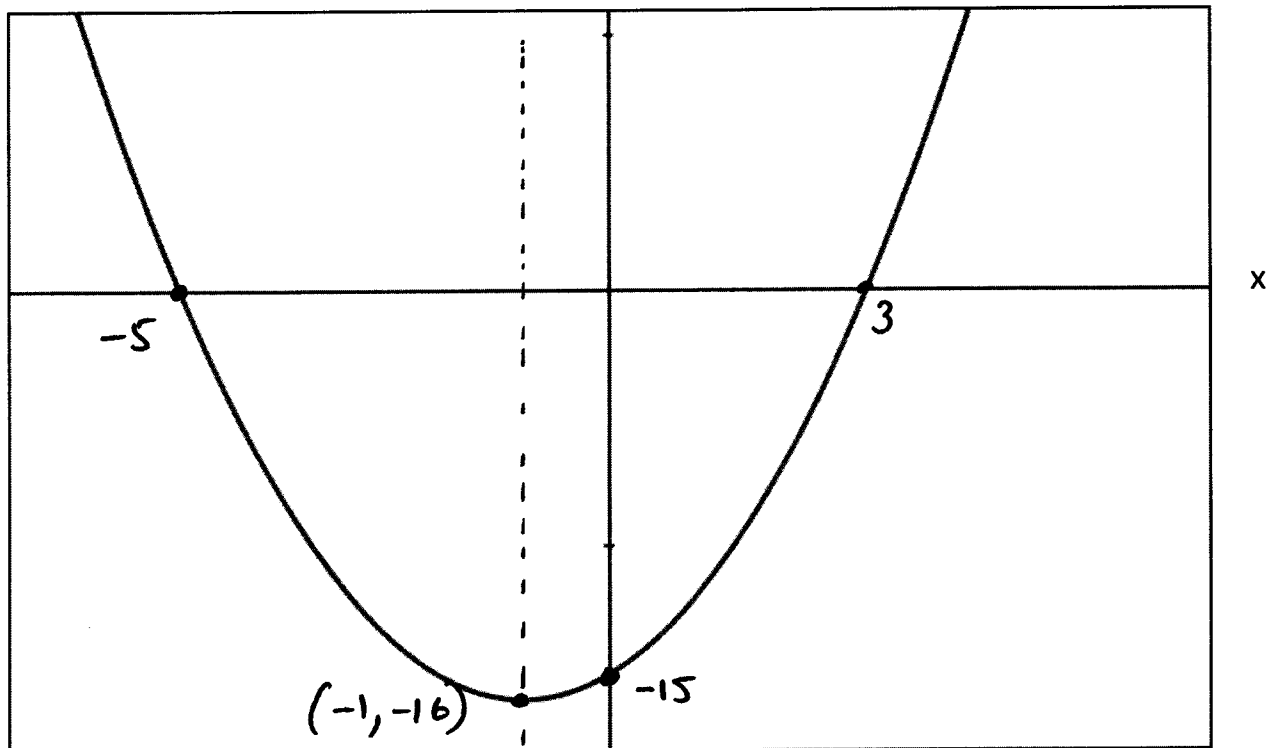
4) Sketch the graph using these 4 points



5) The equation of the line of symmetry of the graph is $x = -1$

(5)

This is the graph $y = x^2 + 2x - 15$



1) Find where the graph crosses

a) The y-axis ($x = 0$) $y = 0^2 + 2 \times 0 - 15 = -15$

b) The x-axis ($y = 0$) $0 = x^2 + 2x - 15$ $x + 5 = 0$ $x = -5$
 $0 = (x + 5)(x - 3)$ $x - 3 = 0$ $x = 3$

2) Find the coordinates of the turning point (Complete the square)

$y = x^2 + 2x - 15$ $y = (x + 1)^2 - 1 - 15$ $y = (x + 1)^2 - 16$
 turning point $(-1, -16)$

3) What is the equation of the line of symmetry?

$x = -1$

4) Solve

a) $x^2 + 2x - 15 = 0$ $x = -5$ or 3

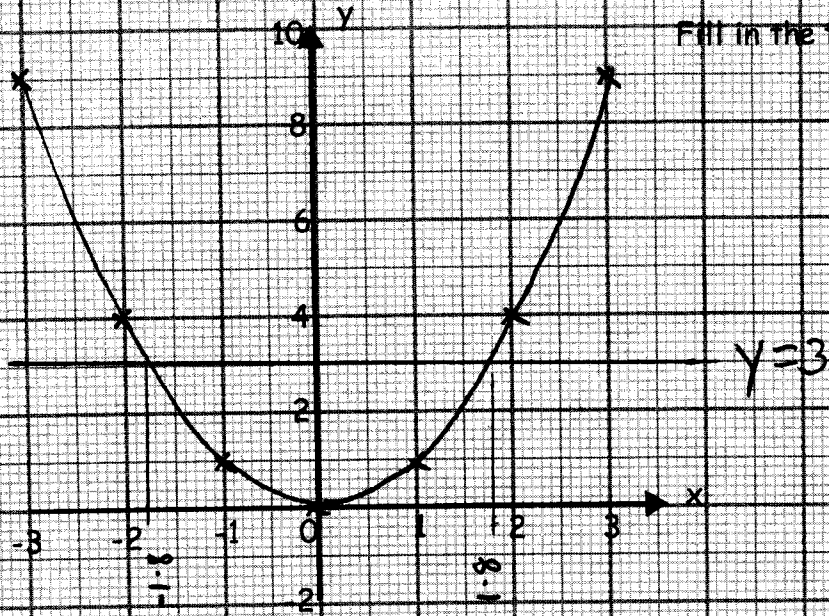
b) $x^2 + 2x - 15 > 0$ $x < -5$ or $x > 3$

c) $x^2 + 2x - 15 \leq 0$ $-5 \leq x \leq 3$

Quadratic Graphs

Fill in the tables and plot the graphs

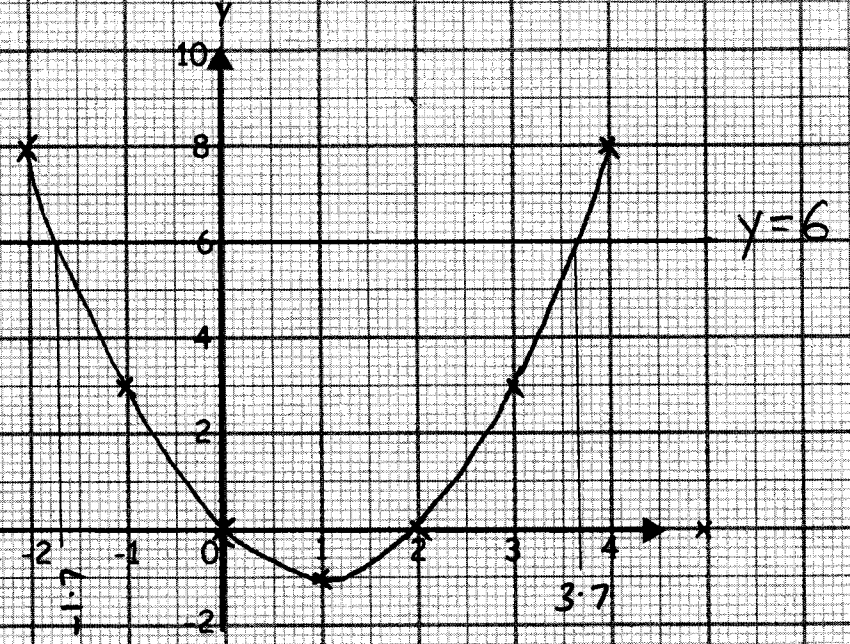
$$y = x^2$$



x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

Use the graph to solve $x^2 = 0$ $x = 0$
 $x^2 = 3$ $x = 1.8$ or -1.8

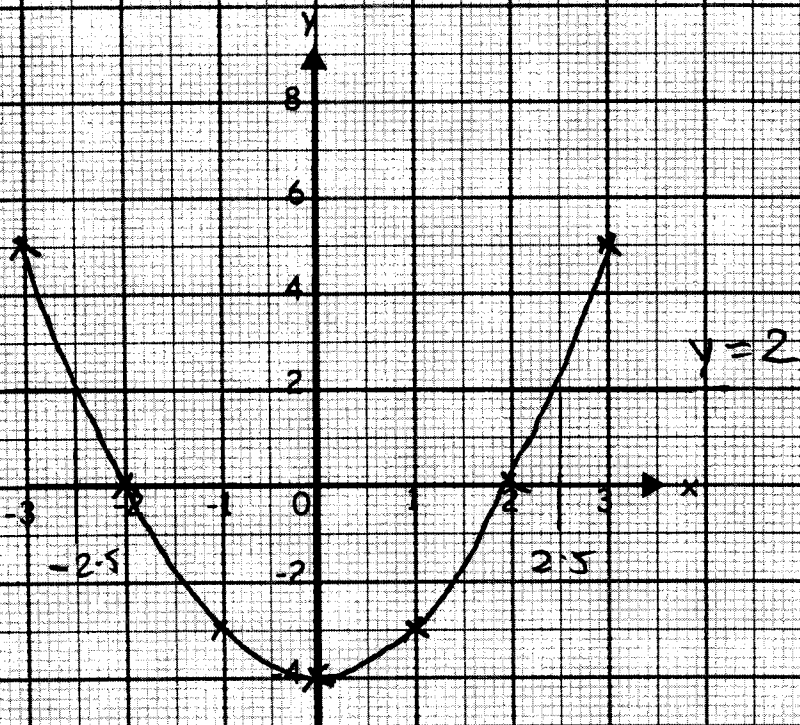
$$y = x^2 - 2x$$



x	-2	-1	0	1	2	3	4
y	8	3	0	-1	0	3	8

Use the graph to solve $x^2 - 2x = 0$ $x = 0$ or 2
 $x^2 - 2x = 6$ $x = 3.7$ or -1.7

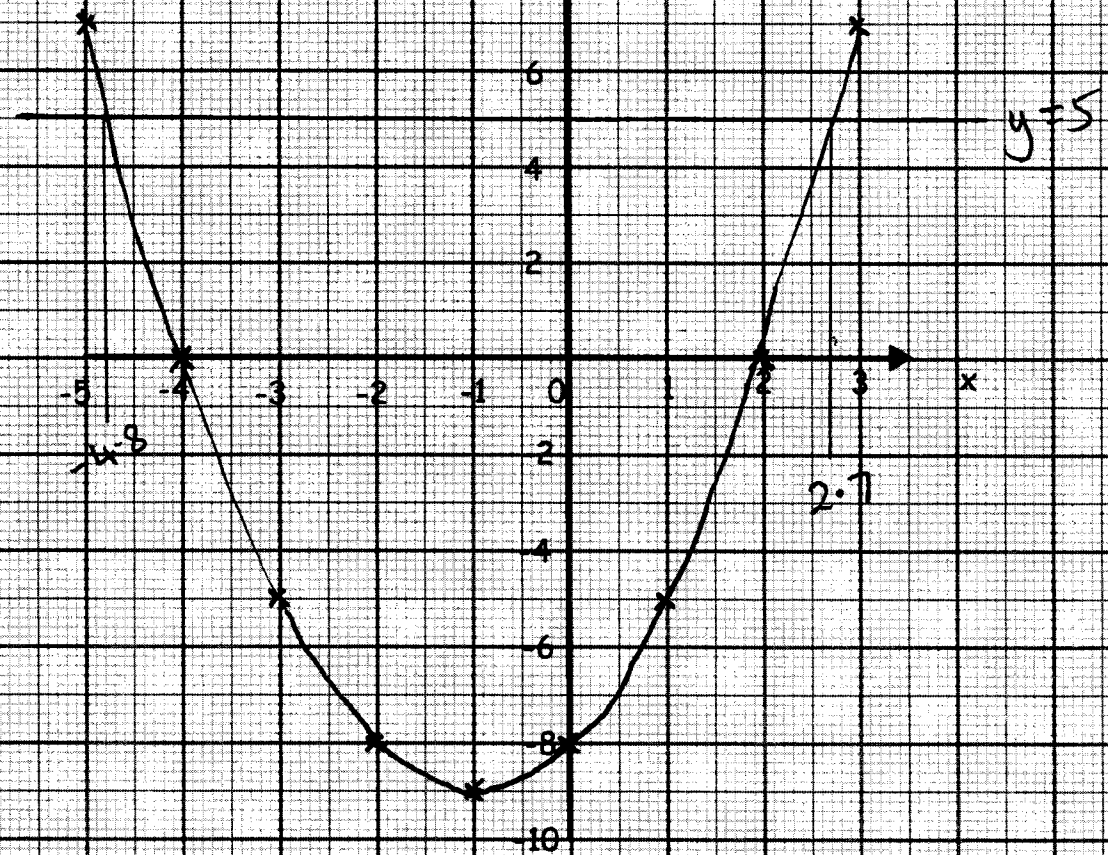
$$y = x^2 - 4$$



x	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5

Use the graph to solve $x^2 - 4 = 0$ $x = 2$ or -2
 $x^2 - 4 = 2$ $x = 2.5$ or -2.5

$$y = x^2 + 2x - 8$$



x	-5	-4	-3	-2	-1	0	1	2	3
y	7	0	-5	-8	-9	-8	-5	0	7

Use the graph to solve $x^2 + 2x - 8 = 0$ $x = -4$ or 2
 $x^2 + 2x - 8 = 5$ $x = 2.7$ or -4.8

(8)

$$y = x^2 - x - 6$$

Fill in the tables and plot the graphs

Use the graph to solve $x^2 - x - 6 = 0$

$$x = -2 \text{ or } 3$$

Use the graph to solve $x^2 - x - 6 = 3$

$$x = 3.5 \text{ or } -2.6$$

What are the coordinates of the turning point?

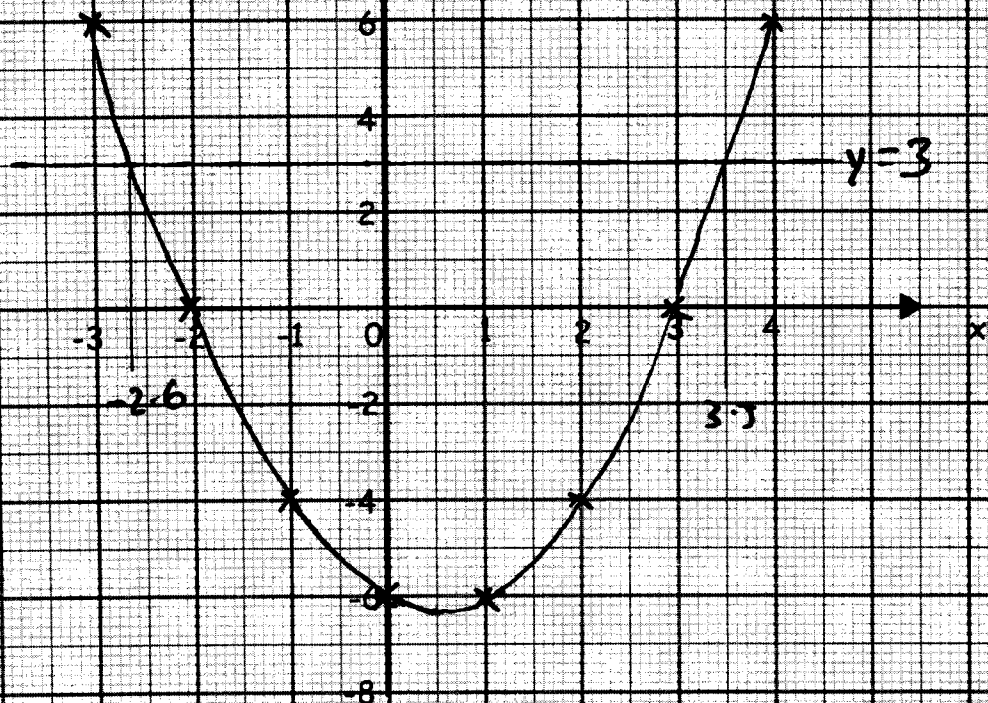
$$\left(\frac{1}{2}, -6.25\right)$$

Use the graph to solve $x^2 - x - 6 \leq 0$

$$-2 \leq x \leq 3$$

Use the graph to solve $x^2 - x - 6 > 0$

$$x < -2 \text{ or } x > 3$$



x	-3	-2	-1	0	1	2	3	4
y	6	0	-4	-6	-6	-4	0	6

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Quadratic Graphs

Hint : when filling in your table work from positive x to negative x. Look for symmetry in the values of y

$$y = 2x + 8 - x^2$$

x	-3	-2	-1	0	1	2	3	4	5
y	-7	0	5	8	9	8	5	0	-7

Plot the points on the graph and join them with a smooth curve.

Use your graph to solve the following equations. (There are nearly always two answer

$$2x + 8 - x^2 = 0$$

Hint: draw the line $y = 0$ find where it crosses your curve

$$x = -2 \text{ or } 4.$$

What is the equation of the line of symmetry of the curve? $x = 1$

What are the coordinates of the minimum/maximum point? $(1, 9)$

$$2x + 8 - x^2 > 0$$

$$-2 < x < 4$$

$$2x + 8 - x^2 \leq 0$$

$$x \leq -2 \text{ or } x \geq 4.$$

Quadratic Graphs

Hint: when filling in your table work from positive x to negative x. Look for symmetry in the values of y

$$y = x^2 - 2x - 8$$

x	-3	-2	-1	0	1	2	3	4	5
y	7	0	-5	-8	-9	-8	-5	0	7

Plot the points on the graph and join them with a smooth curve.

Use your graph to solve the following equations. (There are nearly always two answers)

$x^2 - 2x - 8 = y$ This is the equation of the graph we have already drawn.

$x^2 - 2x - 8 = 0$ Hint: draw the line $y = 0$ find where it crosses your curve $x = -2$ or 4 .

$x^2 - 2x - 8 = -4$ Hint: draw the line $y = -4$ find where it crosses your curve $x = 3.2$ or -1.2

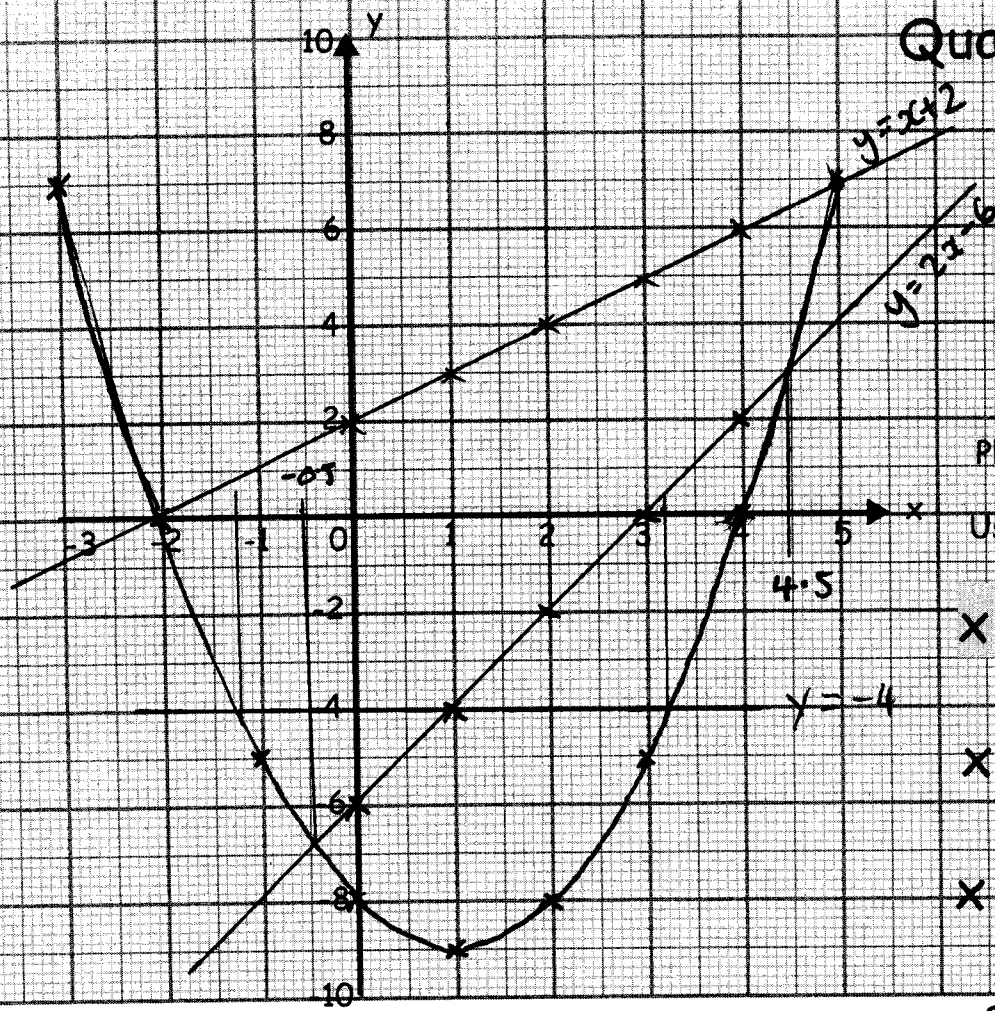
$x^2 - 2x - 8 = 2x - 6$ Hint: draw the line $y = 2x - 6$ find where it crosses your curve $x = 4.5$ or -0.5

$x^2 - 3x - 10 = 0$ Hint: The equation you are given is no longer the equation of the curve you have drawn. You must think how to adapt the equation you are given so that you can use the graph of $x^2 - 2x - 8$ $x = 5$ or -2

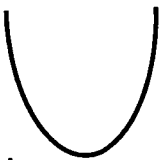
rearrange
 $x^2 - 2x - 8 = x + 2$
 $x^2 - 3x - 10 = 0$
 work at

$x^2 - 2x - 8 = (x^2 - 3x - 10) + (x + 2)$
 equation of graph Equation to solve
 $x^2 - 2x - 8 - x^2 + 3x + 10 = x + 2$
 drawn the line $y = x + 2$

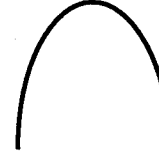
(11)



Remember - Quadratic Graphs



This way if x^2 is positive



This way if x^2 is negative