

Quadratics - Solve equations, Quadratic Formula, Complete the square, Graphs

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Quadratics Equations that factorise

1) Solve $x^2 + 7x + 12 = 0$

If the equation does not equal 0, rearrange it so that it does

Factorise the quadratic $(x + 3)(x + 4) = 0$

This represents two numbers multiplied together so that the answer is 0. For example -

If $x = 1$ $(x + 3)(x + 4)$ becomes $4 \times 5 = 20$ and not 0, so $x = 1$ is not an answer

The only possible way that two numbers multiplied together have an answer of 0 is if one of the numbers is 0.

If $(x + 3)(x + 4) = 0$ Then either $x + 3 = 0, x = -3$ OR $x + 4 = 0, x = -4$

There are two answers, $x = -3$ or -4

2) Solve $x^2 - 4x - 12 = 0$

Factorise $(x - 6)(x + 2) = 0$

Either $x - 6 = 0, x = 6$ OR $x + 2 = 0, x = -2$ Answers $x = 6$ or -2

3) Solve $2x^2 = x + 15$ Rearrange to make 0 $2x^2 - x - 15 = 0$

Factorise $(2x + 5)(x - 3) = 0$

Either $2x + 5 = 0, x = -\frac{5}{2}$ OR $x - 3 = 0, x = 3$ Answers $x = -2.5$ or 3

Solve these quadratic equations by factorising

- | | |
|--|--|
| <p>1) $x^2 + 6x + 5 = 0$
 $(x + 5)(x + 1) = 0$
 $x + 5 = 0 \quad x = -5$
 $x + 1 = 0 \quad x = -1$</p> | <p>6) $2x^2 + x - 21 = 0$ $2x + 7 = 0 \quad x = -\frac{7}{2}$
 $(2x + 7)(x - 3) = 0$ $x - 3 = 0 \quad x = 3$</p> |
| <p>2) $x^2 - 8x - 20 = 0$
 $(x - 10)(x + 2) = 0$
 $x - 10 = 0 \quad x = 10$
 $x + 2 = 0 \quad x = -2$</p> | <p>7) $3x^2 - 11x + 6 = 0$ $3x - 2 = 0 \quad x = \frac{2}{3}$
 $(3x - 2)(x - 3) = 0$ $x - 3 = 0 \quad x = 3$</p> |
| <p>3) $x^2 + x - 20 = 0$
 $(x + 5)(x - 4) = 0$
 $x + 5 = 0 \quad x = -5$
 $x - 4 = 0 \quad x = 4$</p> | <p>8) $2x^2 + 7x - 15 = 0$ $2x - 3 = 0 \quad x = \frac{3}{2}$
 $(2x - 3)(x + 5) = 0$ $x + 5 = 0 \quad x = -5$</p> |
| <p>4) $x^2 - 5x + 6 = 0$
 $(x - 2)(x - 3) = 0$
 $x - 2 = 0 \quad x = 2$
 $x - 3 = 0 \quad x = 3$</p> | <p>9) $3x^2 + 13x + 4 = 0$ $x + 4 = 0 \quad x = -4$
 $(3x + 1)(x + 4) = 0$ $3x + 1 = 0 \quad x = -\frac{1}{3}$</p> |
| <p>5) $x^2 - 25 = 0$
 $(x - 5)(x + 5) = 0$
 $x - 5 = 0 \quad x = 5$ $x + 5 = 0 \quad x = -5$ ①</p> | <p>10) $3x^2 - 11x + 10 = 0$ $3x - 5 = 0 \quad x = \frac{5}{3}$
 $(3x - 5)(x - 2) = 0$ $x - 2 = 0 \quad x = 2$</p> |

The quadratic Formula. The equation must equal ZERO. a is the number of x^2 , b is the number of x's and c is the number part.

PUT ANY NEGATIVE NUMBERS IN BRACKETS

Answers to 2dp

$$ax^2 + bx + c = 0$$

$$a = 1$$

$$x^2 + 4x + 2 = 0$$

$$b = 4$$

$$c = 2$$

$$x = \frac{-\boxed{4} \pm \sqrt{\boxed{4}^2 - 4 \times \boxed{1} \times \boxed{2}}}{2 \times \boxed{1}}$$

$$x = -0.59$$

or

$$x = -3.41$$

$$ax^2 + bx + c = 0$$

$$a = 2$$

$$2x^2 + 5x - 1 = 0$$

$$b = 5$$

$$c = -1$$

$$x = \frac{-\boxed{5} \pm \sqrt{\boxed{5}^2 - 4 \times \boxed{2} \times \boxed{-1}}}{2 \times \boxed{2}}$$

$$x = 0.19$$

or

$$x = -2.69$$

$$ax^2 + bx + c = 0$$

$$a = 4$$

$$4x^2 - 3x - 2 = 0$$

$$b = -3$$

$$c = -2$$

$$x = \frac{-\boxed{-3} \pm \sqrt{\boxed{-3}^2 - 4 \times \boxed{4} \times \boxed{-2}}}{2 \times \boxed{4}}$$

$$x = 1.18$$

or

$$x = -0.43$$

$$2x^2 - 5x = 1$$

$$a = 2$$

rearrange so that it equals zero

$$b = -5$$

$$2x^2 - 5x - 1 = 0$$

$$c = -1$$

$$x = \frac{-\boxed{-5} \pm \sqrt{\boxed{-5}^2 - 4 \times \boxed{2} \times \boxed{-1}}}{2 \times \boxed{2}}$$

$$x = 2.69$$

or

$$x = -0.19$$

②

To solve the quadratic equation $ax^2 + bx + c = 0$ use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

NOTE - the equation you are solving must equal 0 before you read off the values of a, b and c.

	Is the equation in the form $ax^2 + bx + c = 0$? If not rearrange it	taken from the equation			Use the equation to get the two answers. Be careful when substituting negative numbers. Round answers to 2 dp
		a number of x^2	b number of x	c the number	
$x^2 + 5x + 6 = 0$	Yes	1	5	6	$x = -2$ or -3
$x^2 + 7x + 5 = 0$	Yes	1	7	5	$x = -0.81$ or -6.19
$5x^2 + 2x = 4$	No $5x^2 + 2x - 4 = 0$	5	2	-4	$x = 0.72$ or -1.12
$3x^2 = 14x - 5$	No $3x^2 - 14x + 5 = 0$	3	-14	5	$x = 4.28$ or 0.39
$x(9x - 1) = 4$	No $9x^2 - x = 4$ $9x^2 - x - 4 = 0$	9	-1	-4	$x = 0.72$ or -0.61
$7 - 2x^2 = 14x$	No $0 = 2x^2 + 14x - 7$ OR $2x^2 + 14x - 7 = 0$	2	14	-7	$x = 0.47$ or -7.47

③

Completing the square for $1x^2$ only

$x^2 + 4x + 7$ is to be written in the form $(x + a)^2 + b$

$$x^2 + 4x + 7$$

half this number (the number of x 's) this gives this number

Now consider what would happen if you expanded

$$(x + 2)^2 = x^2 + 4x + 4$$

We want $x^2 + 4x$ but don't want the $+4$, so we subtract this

$$(x + 2)^2 - 4 = x^2 + 4x$$

Put this back in the original expression in place of the $x^2 + 4x$

$x^2 + 4x + 7$ becomes

$$(x + 2)^2 - 4 + 7$$

$$(x + 2)^2 + 3$$

So $x^2 + 4x + 7$ is the same as $(x + 2)^2 + 3$

$a = 2$ and $b = 3$

Complete the square on these quadratic expressions

$$\begin{aligned} 1) x^2 + 2x + 9 &= (x+1)^2 - 1 + 9 \\ &= (x+1)^2 + 8 \end{aligned}$$

$$\begin{aligned} 2) x^2 - 2x + 7 &= (x-1)^2 - 1 + 7 \\ &= (x-1)^2 + 6 \end{aligned}$$

$$\begin{aligned} 3) x^2 - 4x + 10 &= (x-2)^2 - 4 + 10 \\ &= (x-2)^2 + 6 \end{aligned}$$

$$\begin{aligned} 4) x^2 + 6x + 10 &= (x+3)^2 - 9 + 10 \\ &= (x+3)^2 + 1 \end{aligned}$$

$$\begin{aligned} 5) x^2 - 6x - 13 &= (x-3)^2 - 9 - 13 \\ &= (x-3)^2 - 22 \end{aligned}$$

Sketch the graph

$$y = x^2 + 2x - 8$$

1) Find where the graph crosses the y axis, this is when $x = 0$

$$y = 0^2 + 2 \times 0 - 8 = -8$$

2) Find where the graph crosses the x axis, this is when $y = 0$

$$0 = x^2 + 2x - 8$$

Factorise and solve

$$(x + 4)(x - 2) = 0 \quad x = -4 \text{ or } 2$$

3) Find the turning point of the graph by completing the square

$$y = x^2 + 2x - 8 = (x + 1)^2 - 1 - 8$$

$$y = (x + 1)^2 - 9$$

The minimum value a number squared can be is 0. $0^2 = 0$

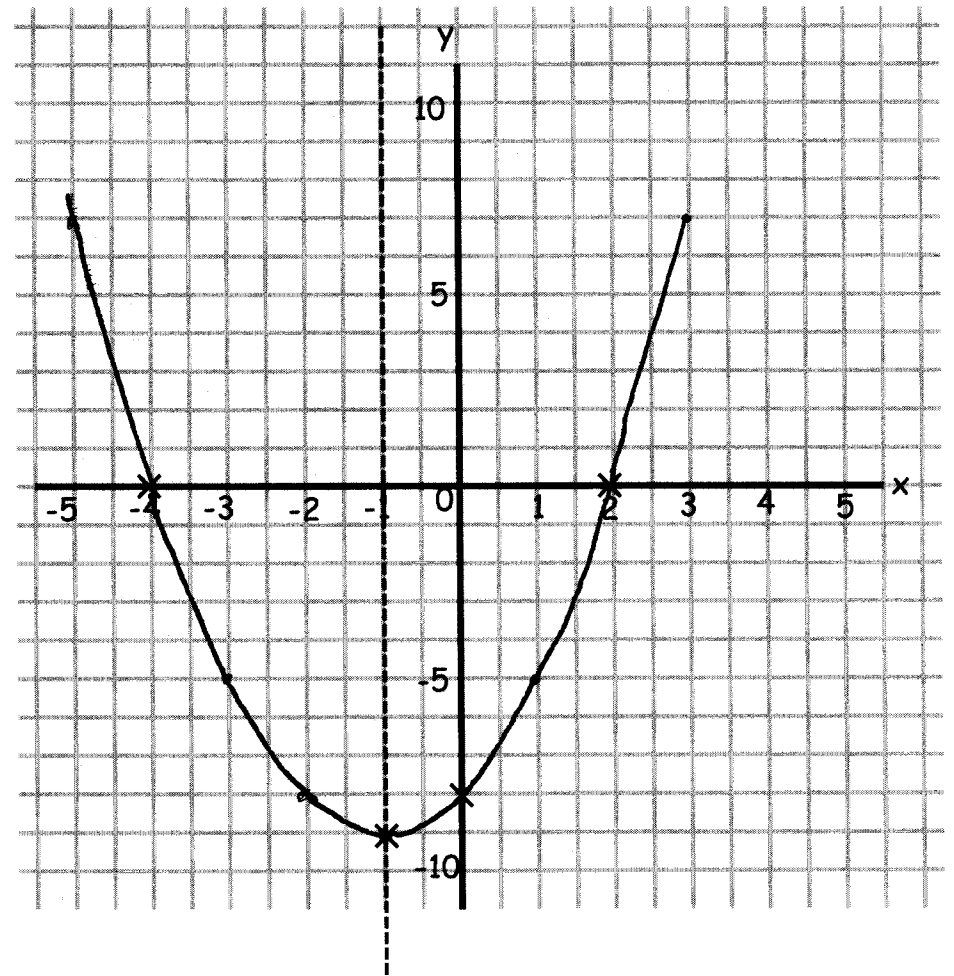
The lowest value y can be is when the squared part is 0.

$$y = 0 - 9 = -9$$

$$\text{so } (x + 1)^2 = 0, \quad x + 1 = 0, \quad x = -1$$

The turning point of the graph is at $(-1, -9)$

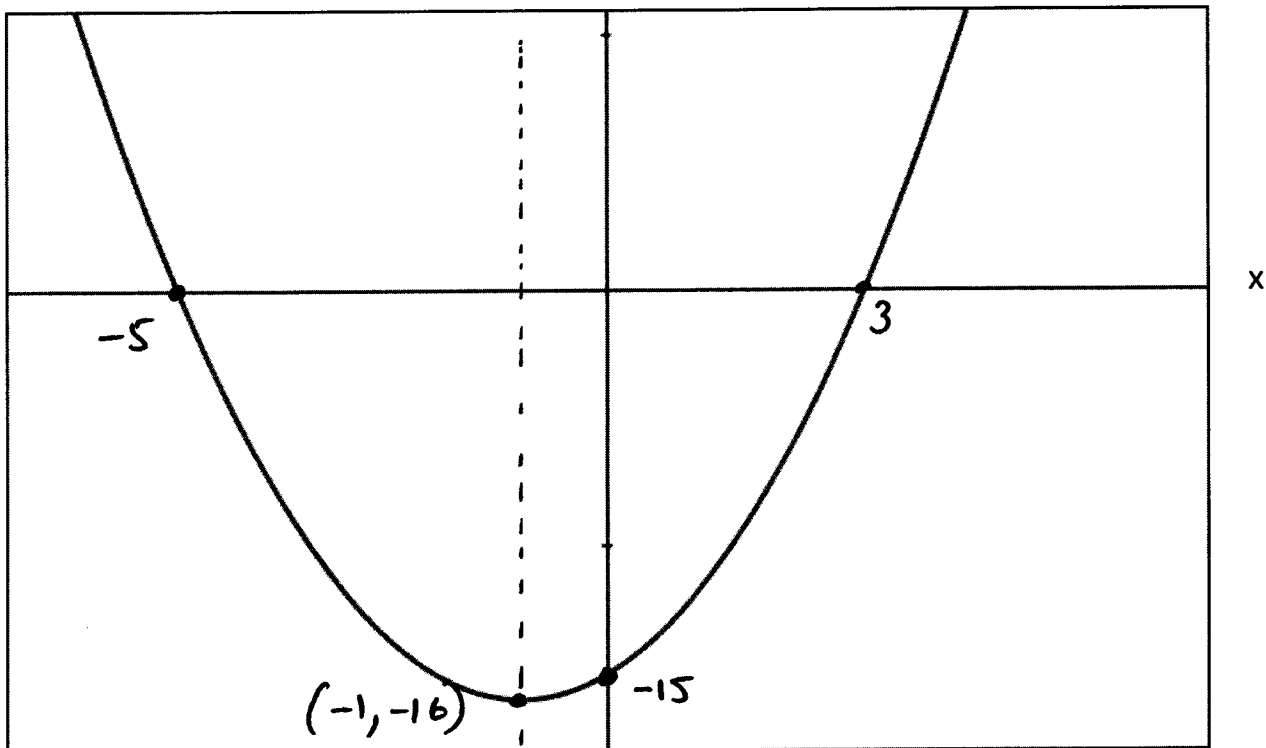
4) Sketch the graph using these 4 points



5) The equation of the line of symmetry of the graph is $x = -1$

(5)

This is the graph $y = x^2 + 2x - 15$



1) Find where the graph crosses

a) The y-axis ($x = 0$) $y = 0^2 + 2 \times 0 - 15 = -15$

b) The x-axis ($y = 0$) $0 = x^2 + 2x - 15$ $x + 5 = 0$ $x = -5$
 $0 = (x + 5)(x - 3)$ $x - 3 = 0$ $x = 3$

2) Find the coordinates of the turning point (Complete the square)

$y = x^2 + 2x - 15$ $y = (x + 1)^2 - 1 - 15$ $y = (x + 1)^2 - 16$
 turning point $(-1, -16)$

3) What is the equation of the line of symmetry?

$x = -1$

4) Solve

a) $x^2 + 2x - 15 = 0$ $x = -5$ or 3

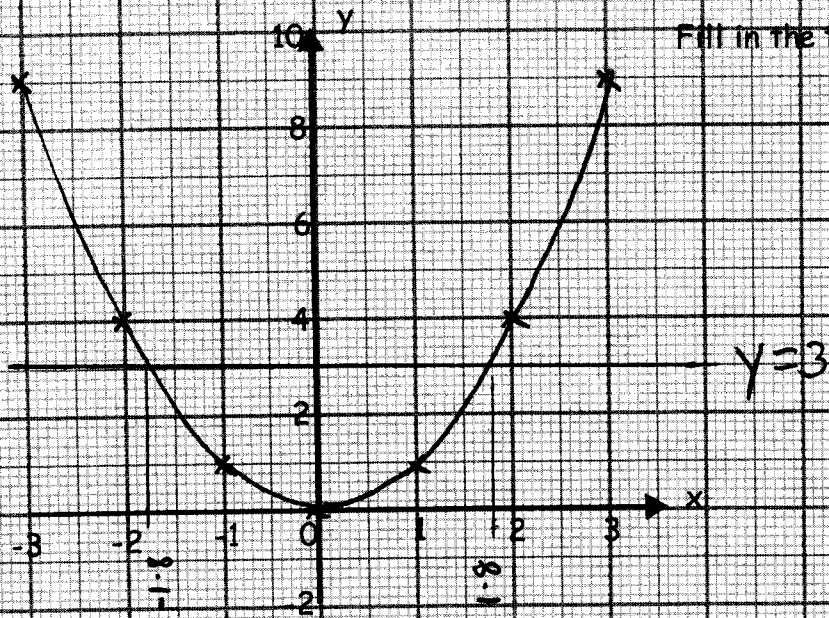
b) $x^2 + 2x - 15 > 0$ $x < -5$ or $x > 3$

c) $x^2 + 2x - 15 \leq 0$ $-5 \leq x \leq 3$

Quadratic Graphs

Fill in the tables and plot the graphs

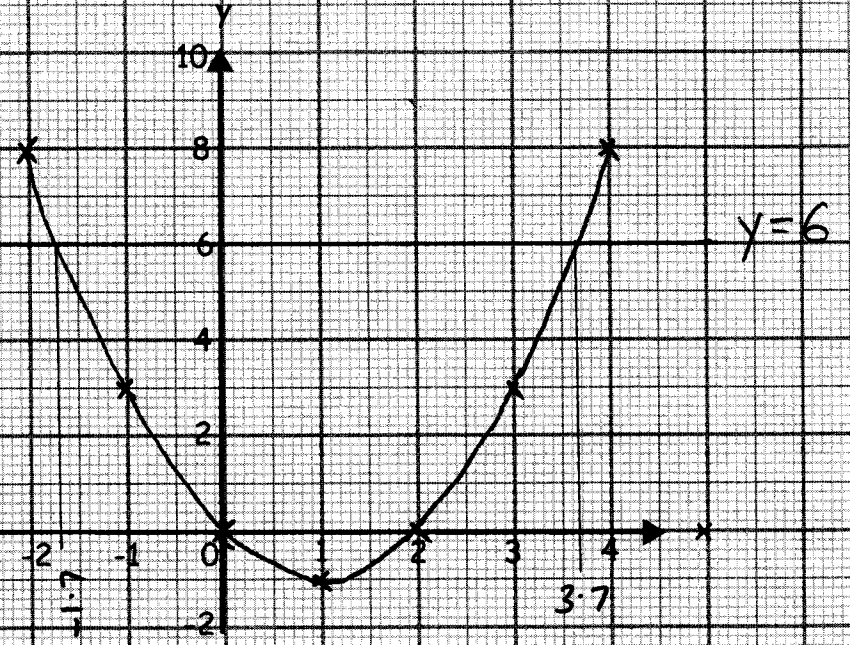
$$y = x^2$$



x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

Use the graph to solve $x^2 = 0$ $x = 0$
 $x^2 = 3$ $x = 1.8$ or -1.8

$$y = x^2 - 2x$$



x	-2	-1	0	1	2	3	4
y	8	3	0	-1	0	3	8

Use the graph to solve $x^2 - 2x = 0$ $x = 0$ or 2
 $x^2 - 2x = 6$ $x = 3.7$ or -1.7