SINE and COSINE RULES. AREA OF A TRIANGLE. EXACT TRIG VALUES. TRIG GRAPHS

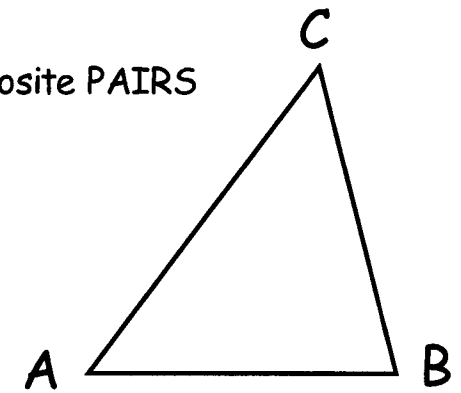# SINE RULE - For Non Right Angled Triangles

Works for TWO opposite PAIRS

1) You have to learn the rule. $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
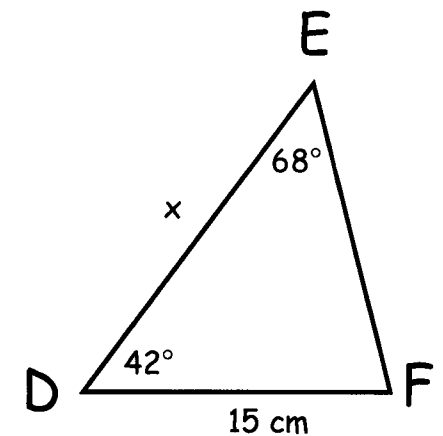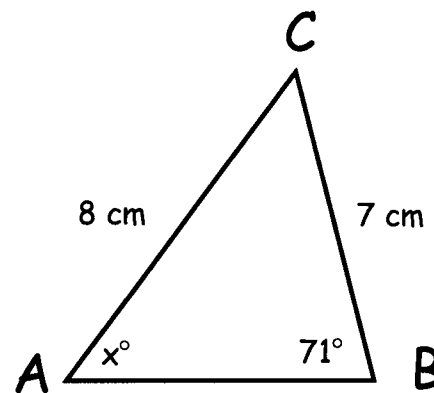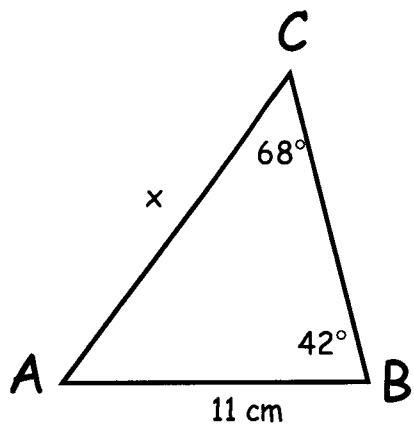
2) Sides are lower case (a,b,c) and angles are upper case (A,B,C). Side "a" is opposite angle "A" and so on.

3) To find ANGLES just flip the whole rule. $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

4) You only need TWO bits from the rule. Choose the TWO bits you need. Eg $$\frac{a}{\sin A} = \frac{c}{\sin C}$$ OR $$\frac{\sin B}{b} = \frac{\sin C}{c}$$

C

A   B

C

68°

x

42°

A   B

11 cm

C

8 cm   7 cm

x°   71°

A   B

E

68°

x

42°

D   F

15 cm

①

COSINE RULE - For Non Right Angled Triangles
Works for THREE SIDES and ONE ANGLE

1) You have to learn the rules. $a^2 = b^2 + c^2 - 2bc \cos A$

2) Sides are lower case (a,b,c) and angles are upper case (A,B,C). Side "a" is opposite angle "A" and so on.

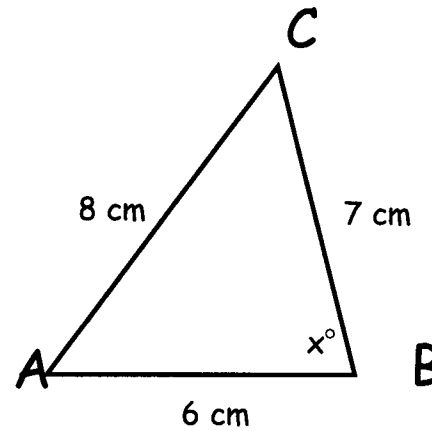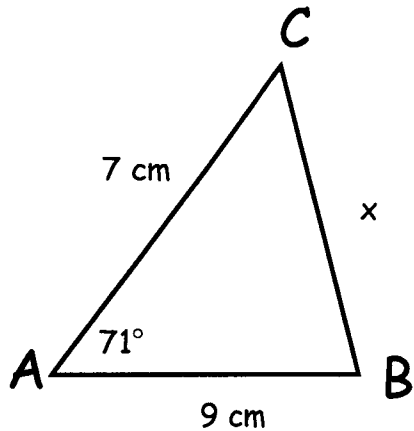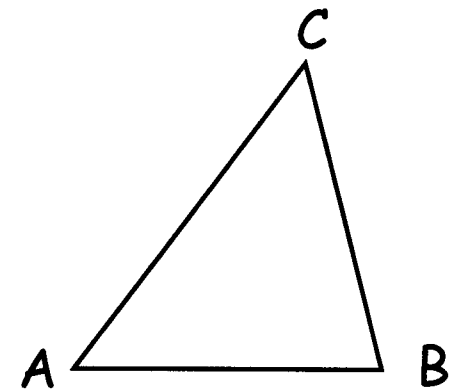3) To find ANGLES you have to rearrange the rule $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

4) If you are not finding side "a" or angle "A" you can either i) SWAP the letters of the triangle so you are finding "a" or "A"

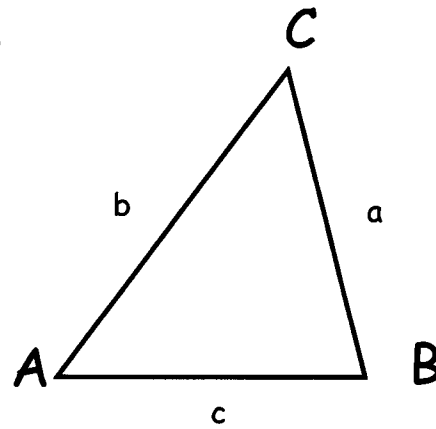ii) Rewrite the rule putting the letters in the place you require eg. $b^2 = a^2 + c^2 - 2ac \cos B$
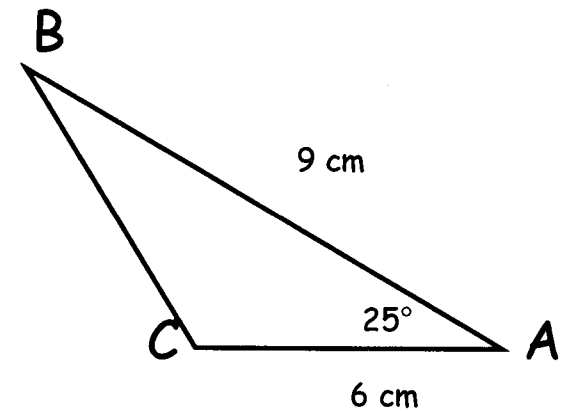$c^2 = a^2 + b^2 - 2ab \cos C$

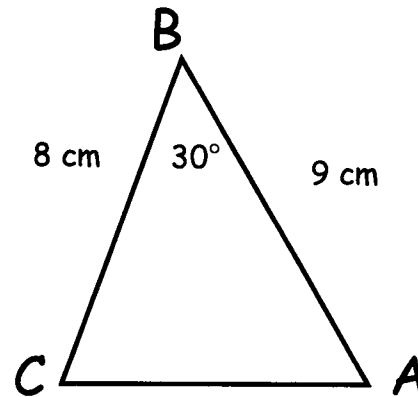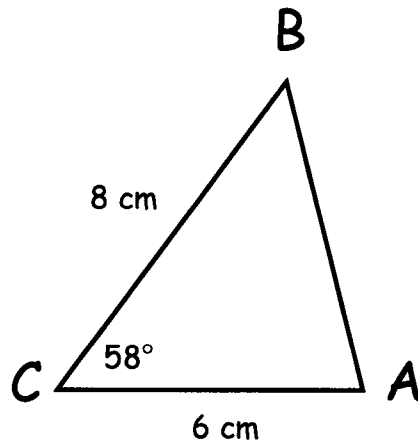$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

②

Area of a Triangle

Area of a triangle $= \frac{1}{2}ab \sin C$

$$= \frac{1}{2}bc \sin A$$

$$= \frac{1}{2}ac \sin B$$

8 cm

58°

6 cm

8 cm   30°   9 cm

9 cm

25°

6 cm

③

Sine and Cosine Rules. Find x

1
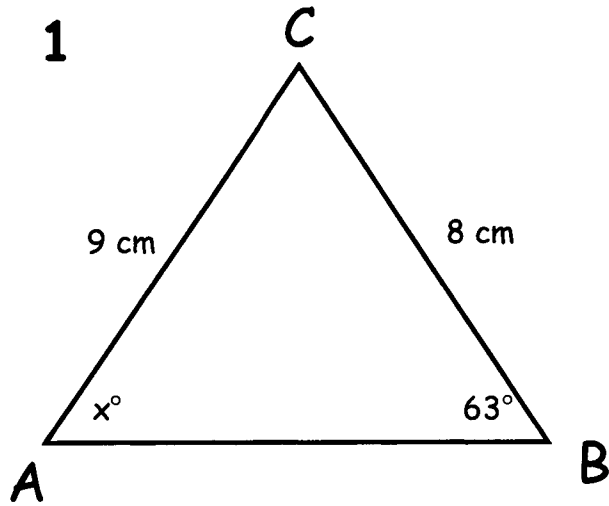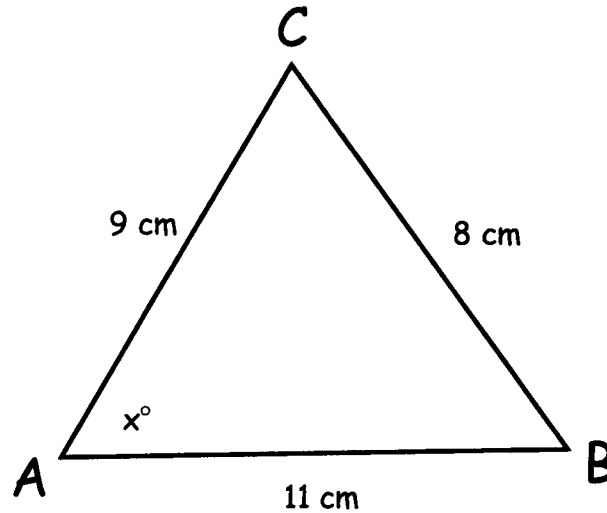
C

9 cm   8 cm

x°   63°

A   B

How many angles are you
given or asked to find?

2

C

9 cm   8 cm

x°

A   B

11 cm

How many angles are you
given or asked to find?

3

C

57°

8 cm

63°

A   B

x

How many angles are you
given or asked to find?

4

C

54°

9 cm   8 cm

A   B
x

How many angles are you
given or asked to find?

5

C

57°

8 cm

63°

A   B
x
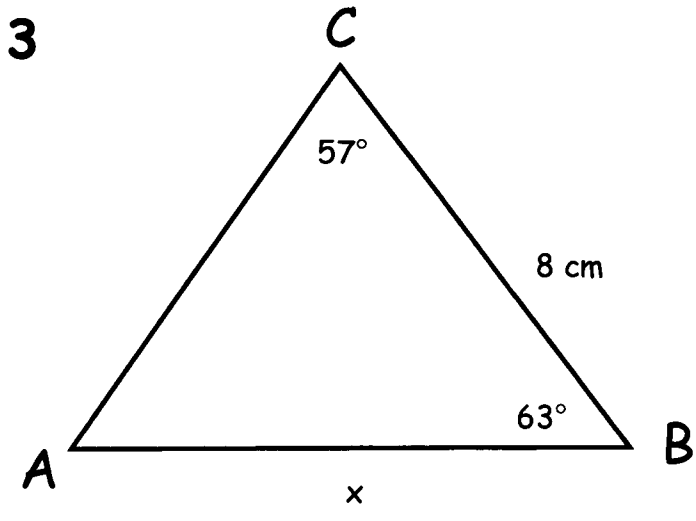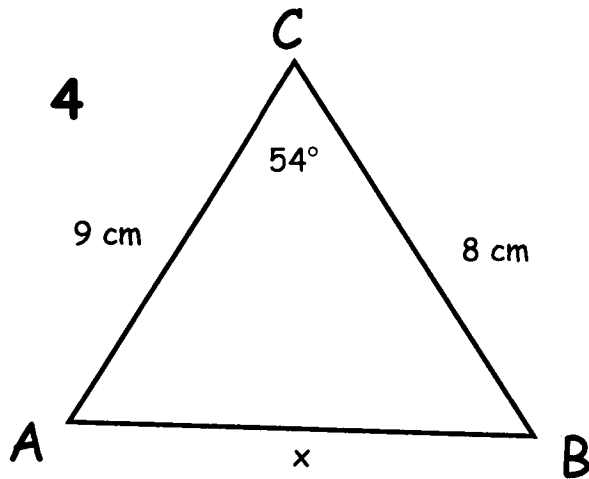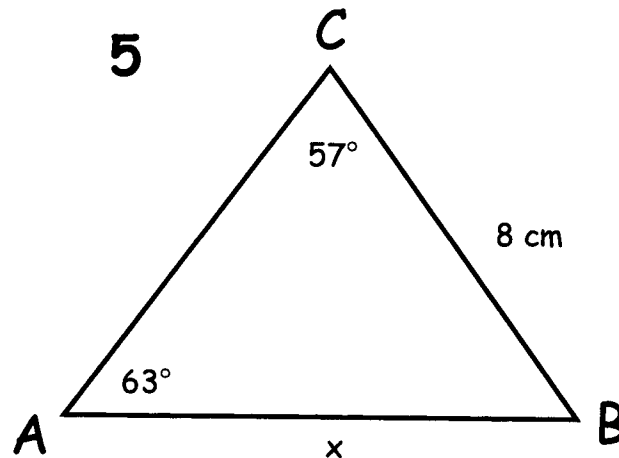
How many angles are you
given or asked to find?

6

E

x°

6 cm   7 cm

F

D
11 cm

How many angles are you
given or asked to find?

④
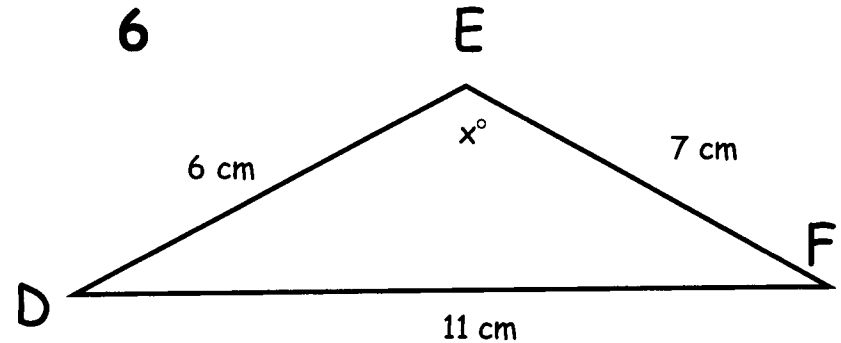
Sine and Cosine rules. Find x and the area of each triangle.

1

9 cm (A to C)
7 cm (C to B)
8 cm (A to B)
x° at A

2

45° at C
7 cm (C to B)
x cm (C to A)
85° at A
B

3

110° at C
5 cm (C to B)
10 cm (A to B)
x° at A

4

12 cm (A to C)
x cm (C to B)
70° at A
14 cm (A to B)

Ⓢ
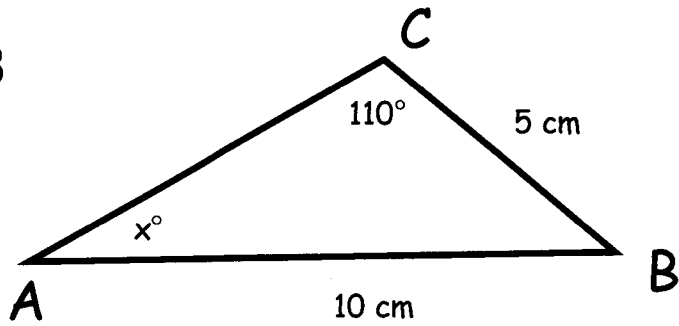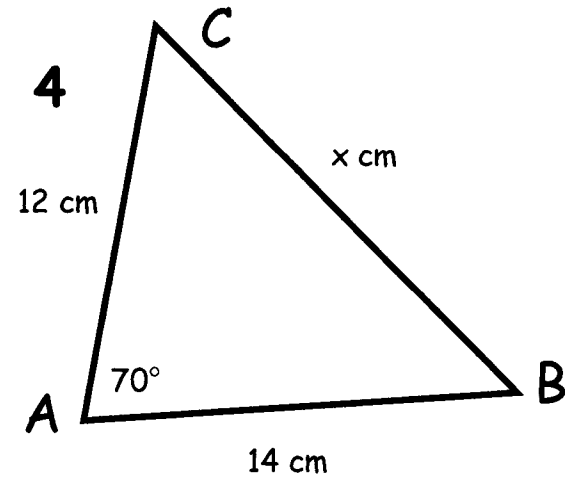
Exact Values for Trigonometry

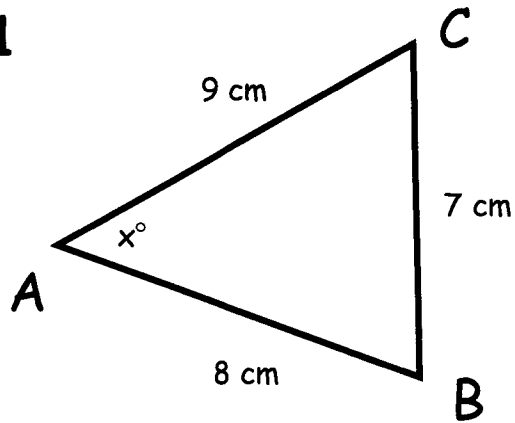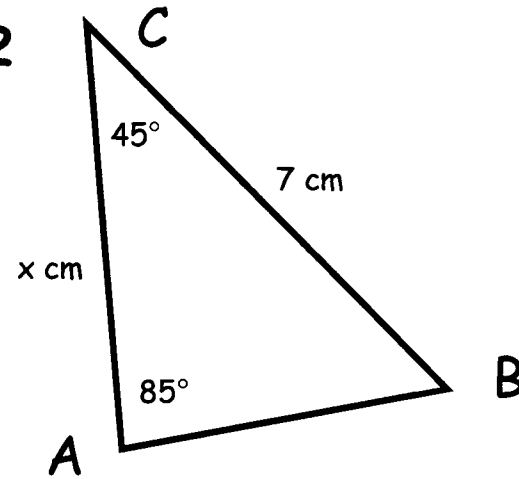| Angle	SIN	COS	TAN
0°	0	1	0
30°	$\dfrac{1}{2}$		
45°			1
60°			
90°	1	0	∞ infinity

You have to remember that $\sin 30 = \dfrac{1}{2}$ and $\tan 45 = 1$

Draw a right angled triangle and label 30, opp and hyp

$\sin 30 = \dfrac{1}{2} = \dfrac{opp}{hyp}$

hyp 2

opp 1

30°
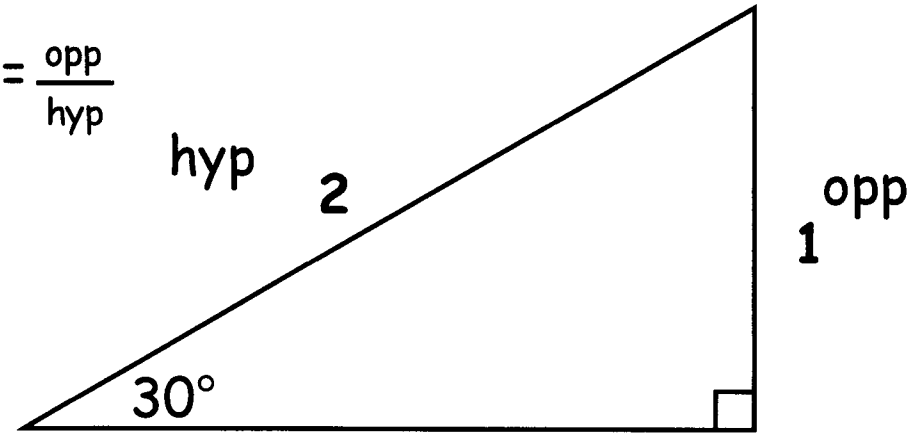
adj Use pythagoras to find the adj

You can now find cos 30 and tan 30. The third angle in the triangle is 60 so you can also find sin 60, cos 60 and tan 60

$\tan 45 = 1 = \dfrac{opp}{adj}$

Use pythagoras to find the hyp hyp

opp 1

You can now find sin 45 and cos 45

45°

1 adj
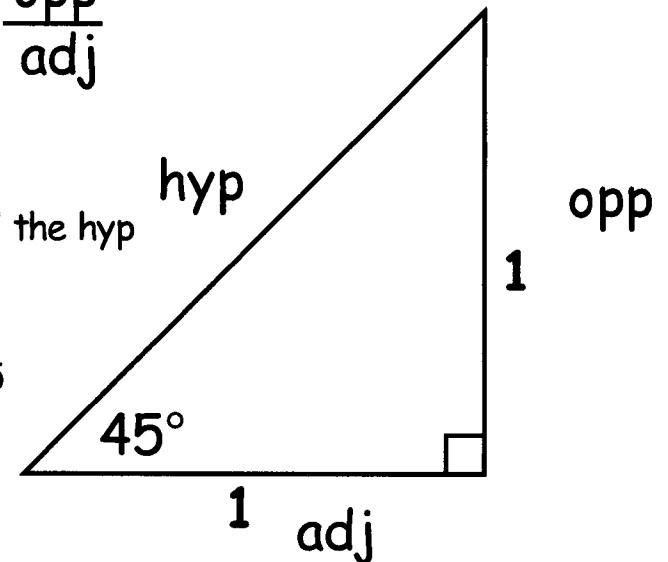
Remember also that over the range 0 to 90

Sin starts with 0 and ends with 1

Cos goes the other way. Starts with 1 and ends in 0

Tan starts with 0 and ends with infinity ∞

⑥

Exact Values for Trigonometry
This is an alternative method for remembering the exact values

The angle headings are 0, 30, 45, 60 and 90. Each one is a fraction.

for SIN — the numerator is the numbers 0 to 4 square rooted
the denominator is 2

for COS — the numerator is the numbers 4 to 0 square rooted
the denominator is 2

for TAN — the numerator is the numbers 0 to 4 square rooted
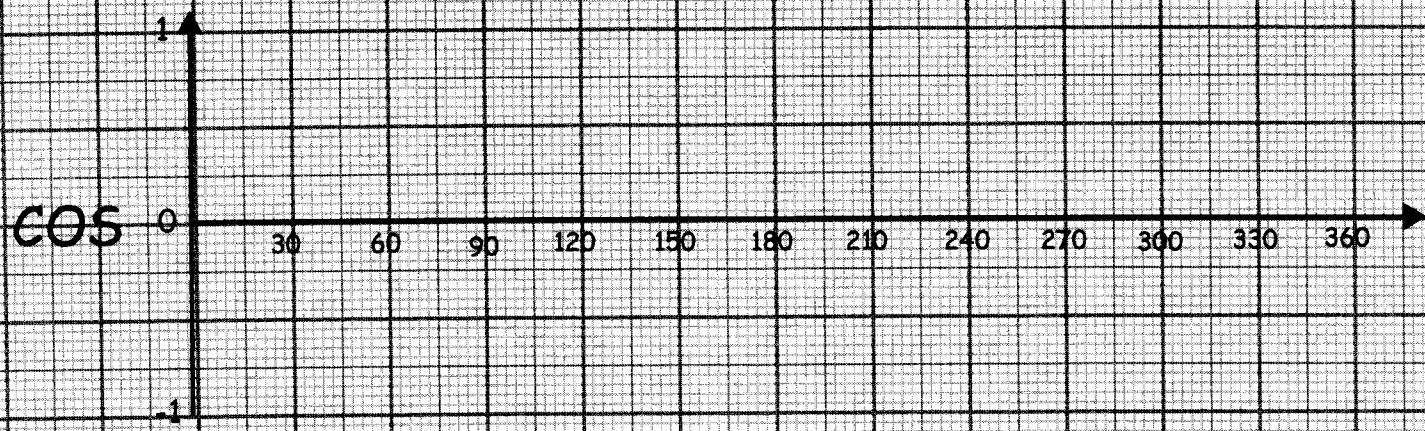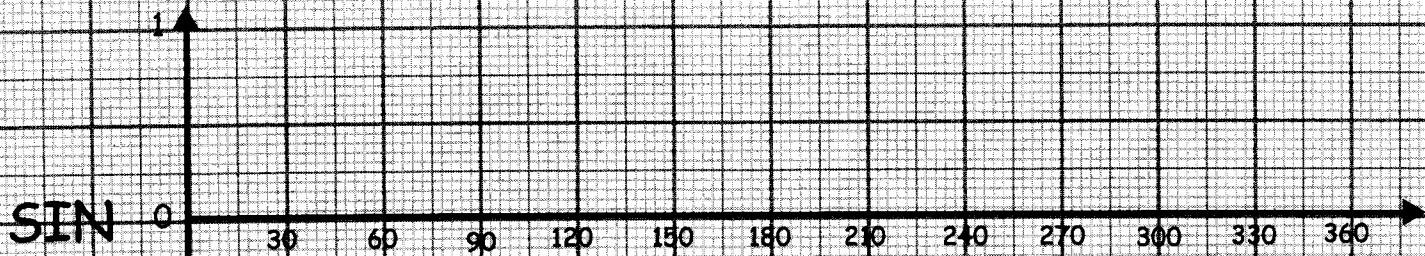the denominator is the numbers 4 to 0 square rooted

Examples

$$\sin 30 = \frac{\sqrt{1}}{2} = \frac{1}{2} \qquad \cos 45 = \frac{\sqrt{2}}{2}$$

$$\tan 30 \quad \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

0° 30° 45° 60° 90°

$$\sin \quad \frac{\sqrt{0 \quad 1 \quad 2 \quad 3 \quad 4}}{2}$$

$$\cos \quad \frac{\sqrt{4 \quad 3 \quad 2 \quad 1 \quad 0}}{2}$$

$$\tan \quad \frac{\sqrt{0 \quad 1 \quad 2 \quad 3 \quad 4}}{\sqrt{4 \quad 3 \quad 2 \quad 1 \quad 0}}$$

Using this method complete the table

Angle	SIN	COS	TAN
0°			
30°			
45°			
60°			
90°			

⑦

SIN

0   30   60   90   120   150   180   210   240   270   300   330   360

COS

0   30   60   90   120   150   180   210   240   270   300   330   360

⑧

TAN

30 60 90 120 150 180 210 240 270 300 330 360

⑨