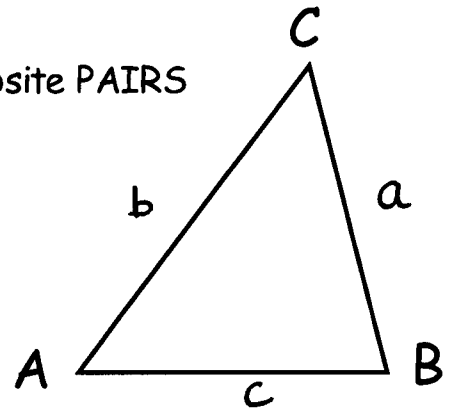
SINE and COSINE RULES. AREA OF A TRIANGLE. EXACT TRIG VALUES. TRIG GRAPHS

SINE RULE - For Non Right Angled Triangles

Works for TWO opposite PAIRS

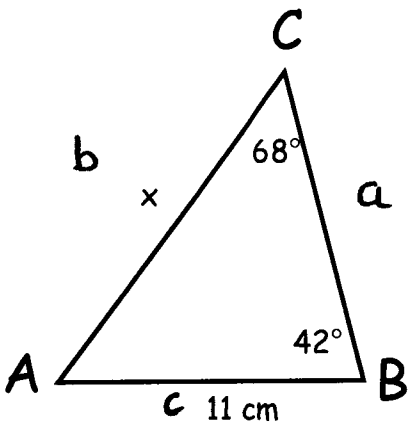


1) You have to learn the rule. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

2) Sides are lower case (a,b,c) and angles are upper case (A,B,C). Side "a" is opposite angle "A" and so on.

3) To find ANGLES just flip the whole rule. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

4) You only need TWO bits from the rule. Choose the TWO bits you need. Eg $\frac{a}{\sin A} = \frac{c}{\sin C}$ OR $\frac{\sin B}{b} = \frac{\sin C}{c}$



$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{x}{\sin 42} = \frac{11}{\sin 68}$

$x = \frac{11 \times \sin 42}{\sin 68}$

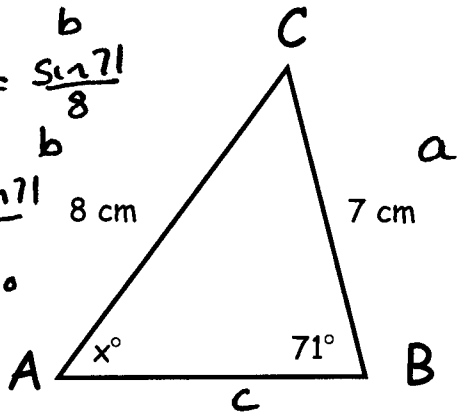
$x = 7.9\text{ cm}$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin x}{7} = \frac{\sin 71}{8}$

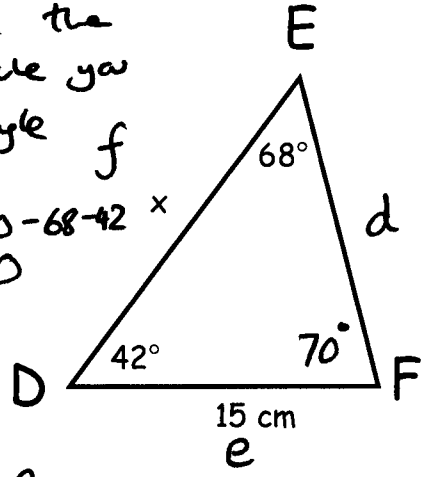
$\sin x = \frac{7 \times \sin 71}{8}$

$x = 55.8°$



To use the sine rule you need angle F.

Angle F = 180 − 68 − 42 = 70



$\frac{f}{\sin F} = \frac{e}{\sin E}$

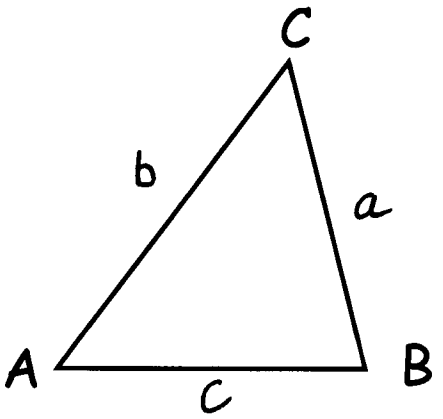
$\frac{x}{\sin 70} = \frac{15}{\sin 68}$

$x = \frac{15 \times \sin 70}{\sin 68}$

$x = 15.2\text{ cm}$

COSINE RULE - For Non Right Angled Triangles

Works for THREE SIDES and ONE ANGLE



1) You have to learn the rules. $a^2 = b^2 + c^2 - 2bc \cos A$

2) Sides are lower case (a,b,c) and angles are upper case (A,B,C). Side "a" is opposite angle "A" and so on.

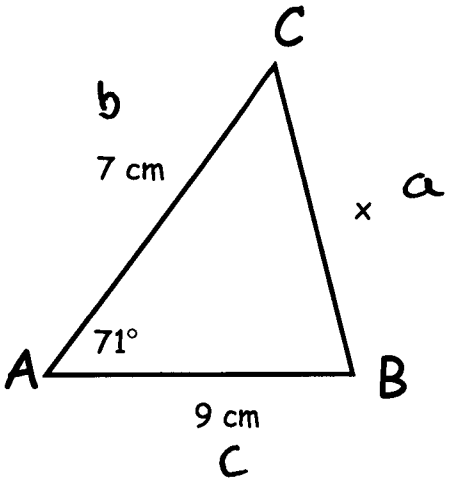
3) To find ANGLES you have to rearrange the rule $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

4) If you are not finding side "a" or angle "A" you can either i) SWAP the letters of the triangle so you are finding "a" or "A"

ii) Rewrite the rule putting the letters in the place you require eg. $b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



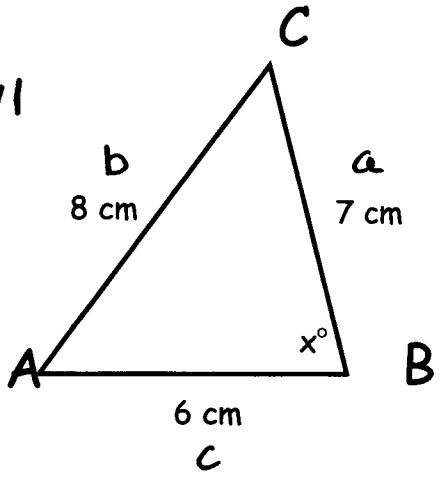
$a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 7^2 + 9^2 - 2 \times 7 \times 9 \times \cos 71$

$x^2 = 88.978$

$x = \sqrt{88.978}$

$x = 9.4$ cm



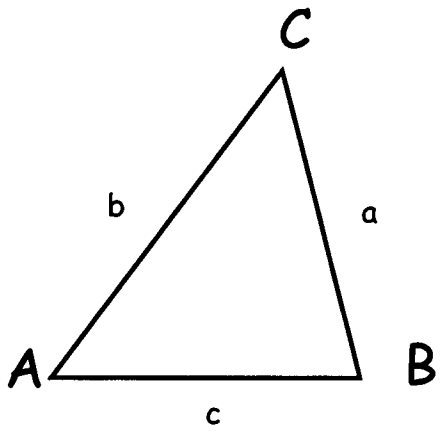
$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{7^2 + 6^2 - 8^2}{2 \times 7 \times 6}$

$\cos B = 0.25$

$B = 75.5°$

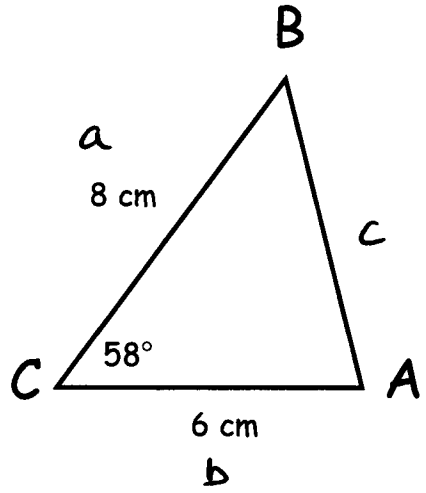
Area of a Triangle



Area of a triangle $= \frac{1}{2}ab \sin C$

$= \frac{1}{2}bc \sin A$

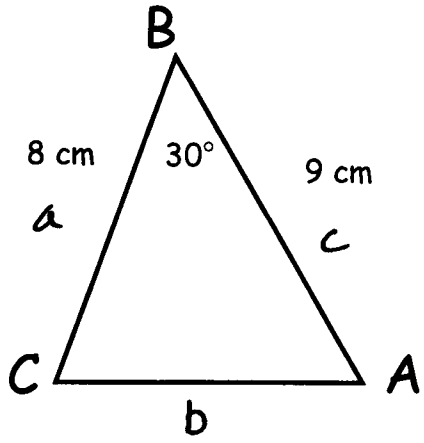
$= \frac{1}{2}ac \sin B$



Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 8 \times 6 \times \sin 58$

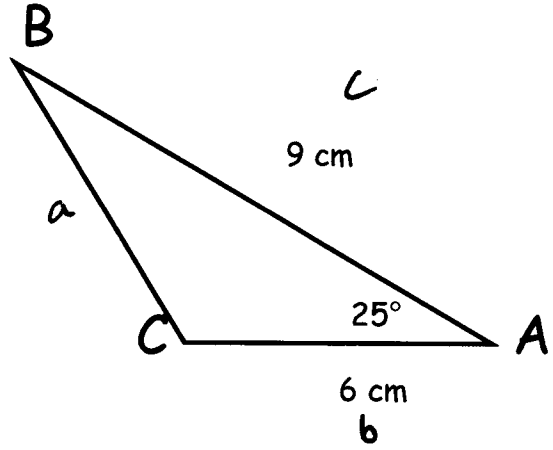
$= 20.4 \text{ cm}^2$



Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 8 \times 9 \times \sin 30$

$= 18 \text{cm}^2$



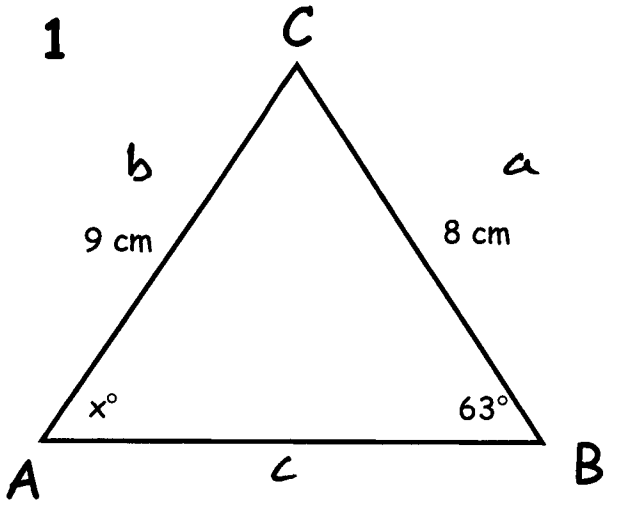
Area $= \frac{1}{2} bc \sin A$

$= \frac{1}{2} \times 6 \times 9 \times \sin 25$

$= 11.4 \text{ cm}^2$

Sine and Cosine Rules. Find x

1

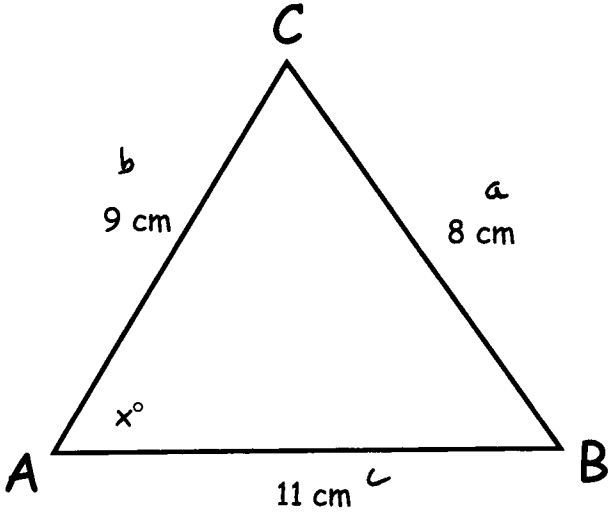


How many angles are you given or asked to find? 2

(Sine rule) $\frac{\sin x}{8} = \frac{\sin 63}{9}$

$x = 52.4°$

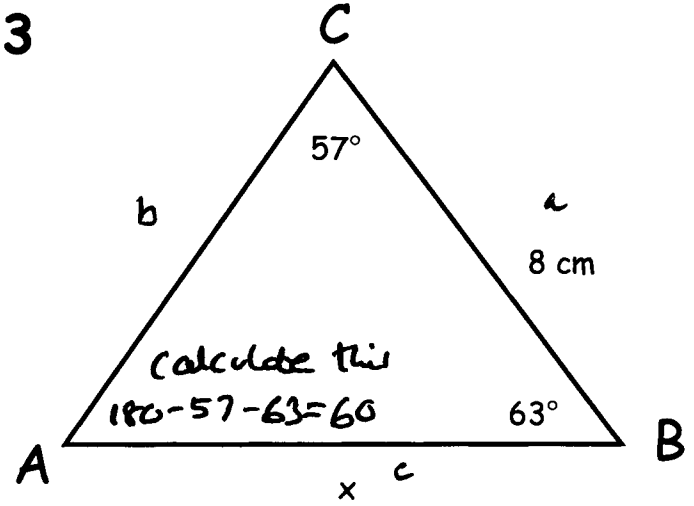
2



How many angles are you given or asked to find? 1 (cosine rule)

$\cos x = \frac{9^2+11^2-8^2}{2\times 9\times 11}$ $x = 45.8°$

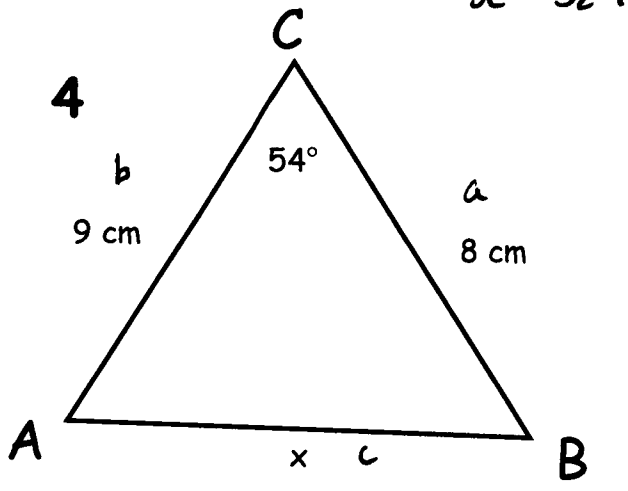
3



How many angles are you given or asked to find? 2 (sine)

$\frac{x}{\sin 57} = \frac{8}{\sin 60}$ $x = 7.7\text{cm}$

4

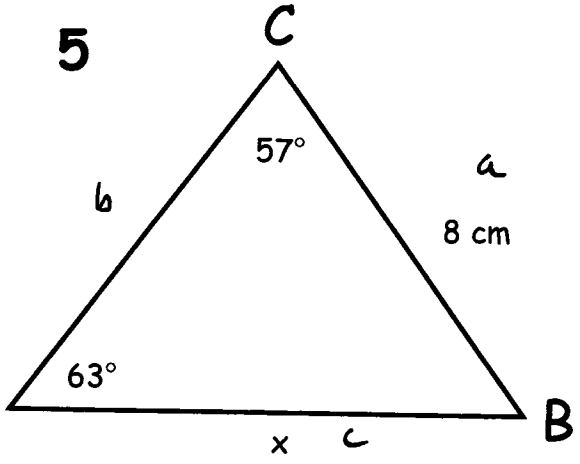


How many angles are you given or asked to find? 1 (cosine)

$x^2 = 9^2+8^2-2\times 9\times 8\times \cos 54$

$x = 7.8\text{cm}$

5

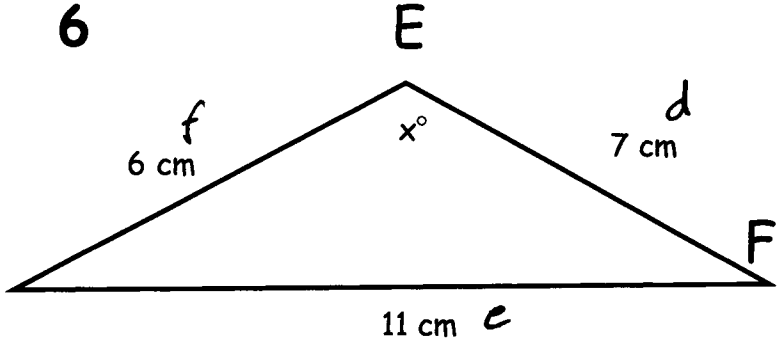


How many angles are you given or asked to find? 2 (Sine)

$\frac{x}{\sin 57} = \frac{8}{\sin 63}$

$x = 7.5\text{cm}$

6



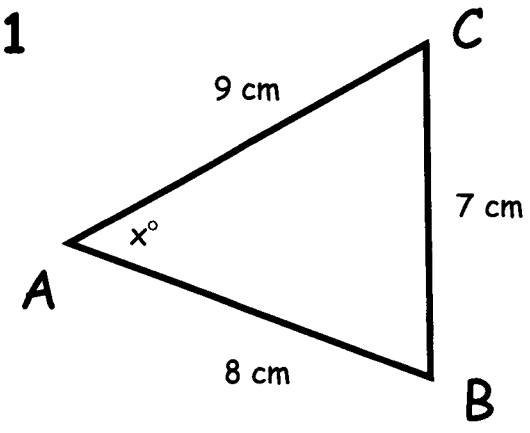
How many angles are you given or asked to find? 1 cosine.

$\cos x = \frac{6^2+7^2-11^2}{2\times 6\times 7}$

$x = 115.4°$

Sine and Cosine rules. Find x and the area of each triangle.

1



Cosine rule.

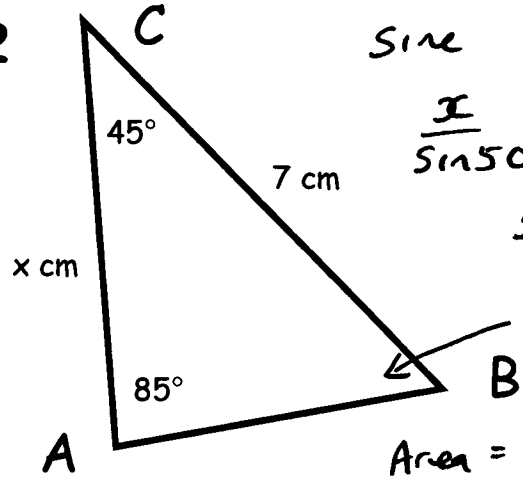
$\cos x = \frac{8^2+9^2-7^2}{2\times8\times9}$

$x = 48.2°$

Area $= \frac{1}{2} \times 8 \times 9 \times \sin 48.2$ (Sin)

$= 26.8\ cm^2$

2



Sine rule.

$\frac{x}{\sin 50} = \frac{7}{\sin 85}$

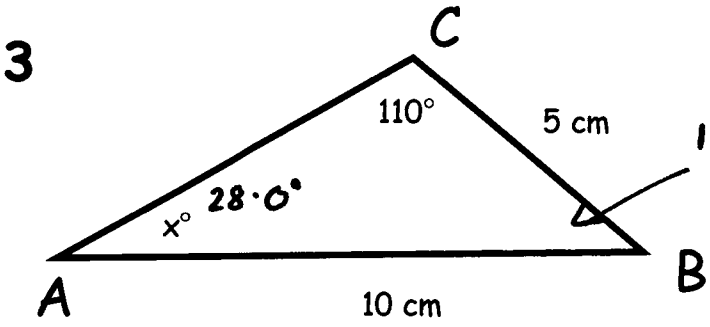
$x = 5.4\ cm$

$180 - 45 - 85 = 50°$

Area $= \frac{1}{2} \times 5.4 \times 7 \times \sin 45$

$= 13.4\ cm^2$

3



$180 - 110 - 28.0° = 42.0°$

Sine rule.

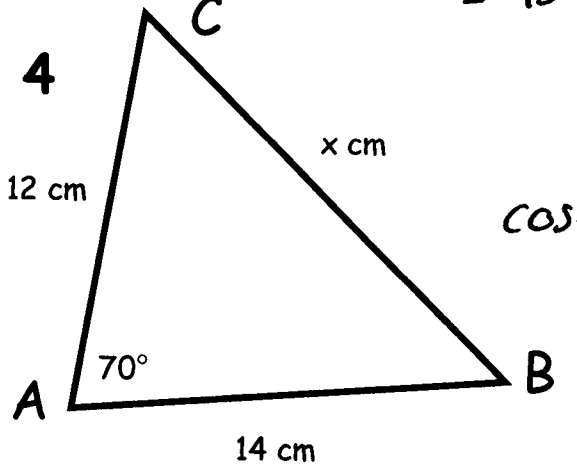
$\frac{\sin x}{5} = \frac{\sin 110}{10}$

$x = 28.0°$

Area $= \frac{1}{2} \times 10 \times 5 \times \sin 42.0$

$= 16.7\ cm^2$

4



Cosine rule.

$x^2 = 12^2 + 14^2 - 2\times12\times14\times\cos 70$

$x = 15.0\ cm$

Area $= \frac{1}{2} \times 12 \times 14 \times \sin 70$

$= 78.9\ cm^2$

Exact Values for Trigonometry

Angle	SIN	COS	TAN
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1} = \sqrt{3}$
90°	1	0	∞ infinity

Remember also that over the range 0 to 90

Sin starts with 0 and ends with 1

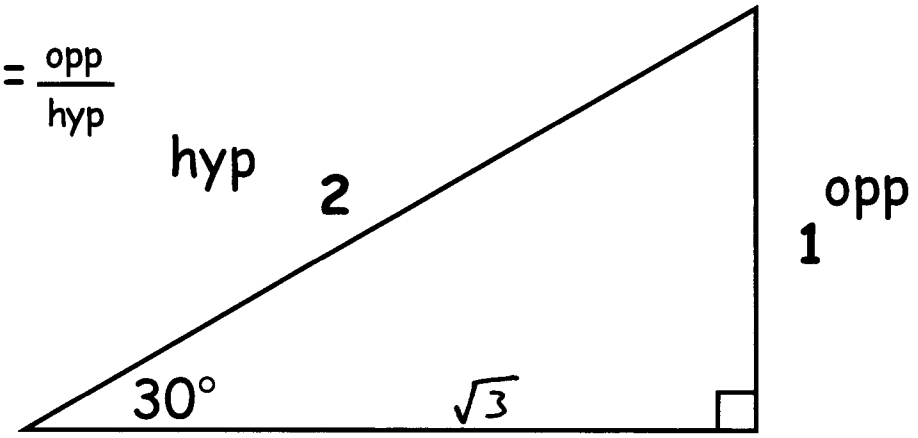
Cos goes the other way. Starts with 1 and ends in 0

Tan starts with 0 and ends with infinity ∞

You have to remember that $\sin 30 = \frac{1}{2}$ and $\tan 45 = 1$

Draw a right angled triangle and label 30, opp and hyp

$\sin 30 = \frac{1}{2} = \frac{\text{opp}}{\text{hyp}}$

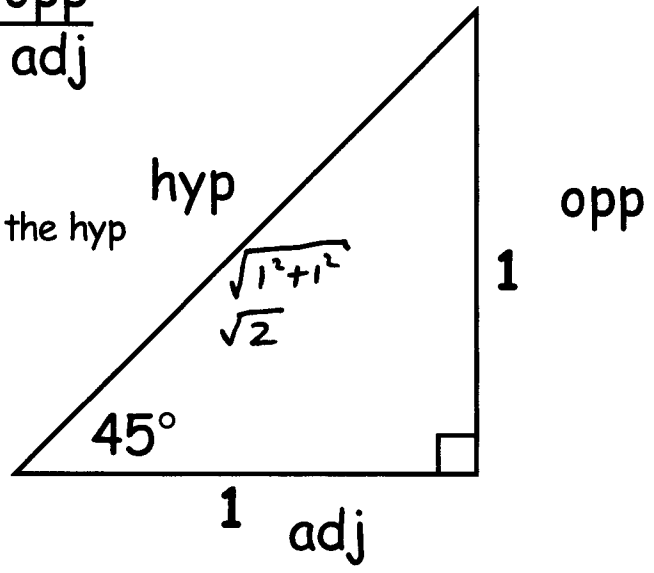


adj Use pythagoras to find the adj $\sqrt{2^2-1^2} = \sqrt{3}$

You can now find cos 30 and tan 30. The third angle in the triangle is 60 so you can also find sin 60, cos 60 and tan 60

$\tan 45 = 1 = \frac{\text{opp}}{\text{adj}}$

Use pythagoras to find the hyp



You can now find sin 45 and cos 45

Exact Values for Trigonometry

This is an alternative method for remembering the exact values

The angle headings are 0, 30, 45, 60 and 90. Each one is a fraction.

for SIN — the numerator is the numbers 0 to 4 square rooted
the denominator is 2

for COS — the numerator is the numbers 4 to 0 square rooted
the denominator is 2

for TAN — the numerator is the numbers 0 to 4 square rooted
the denominator is the numbers 4 to 0 square rooted

	0°	30°	45°	60°	90°
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
tan	$\frac{\sqrt{0}}{\sqrt{4}}$	$\frac{\sqrt{1}}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{1}}$	$\frac{\sqrt{4}}{\sqrt{0}}$

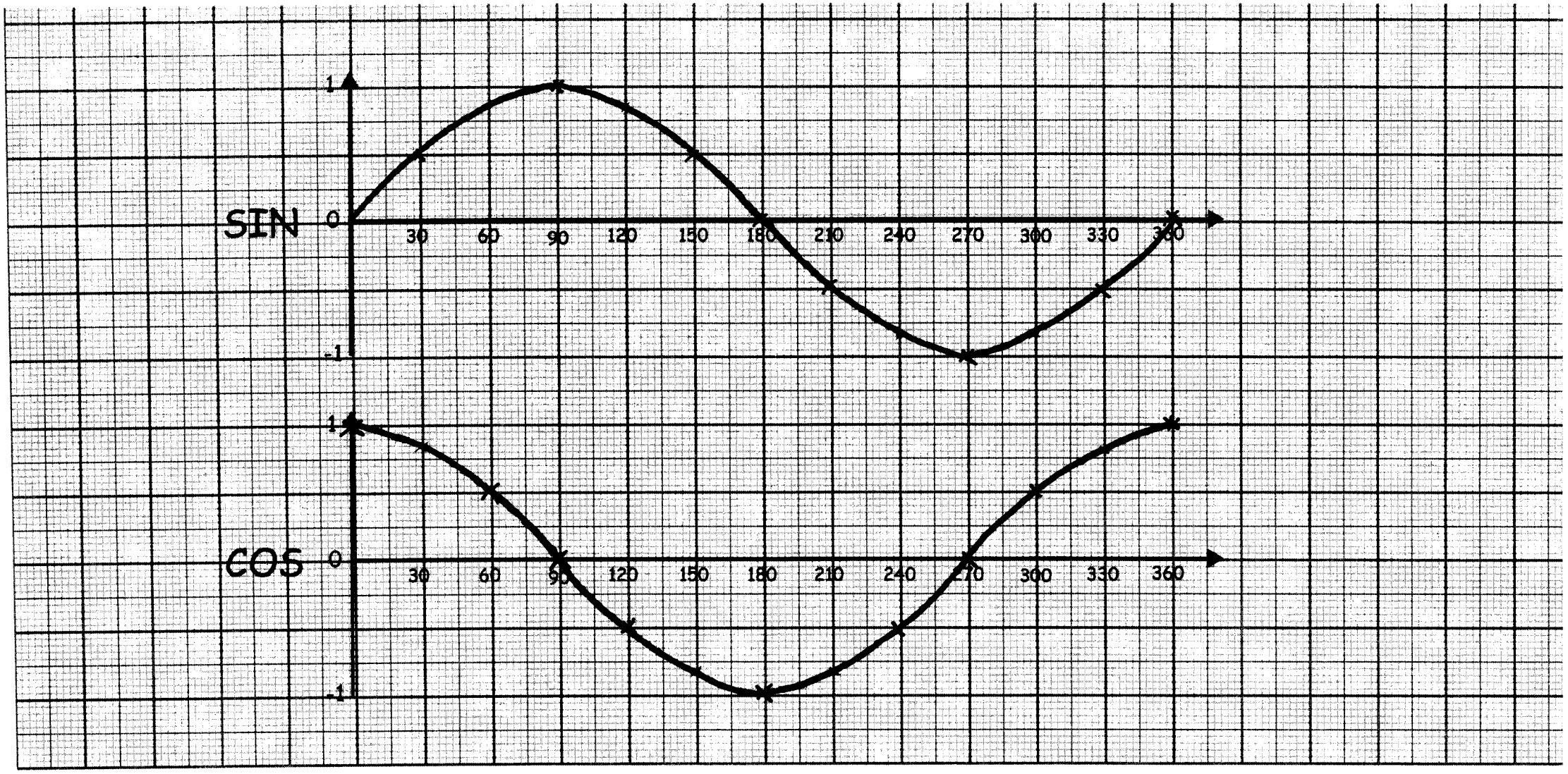
Examples

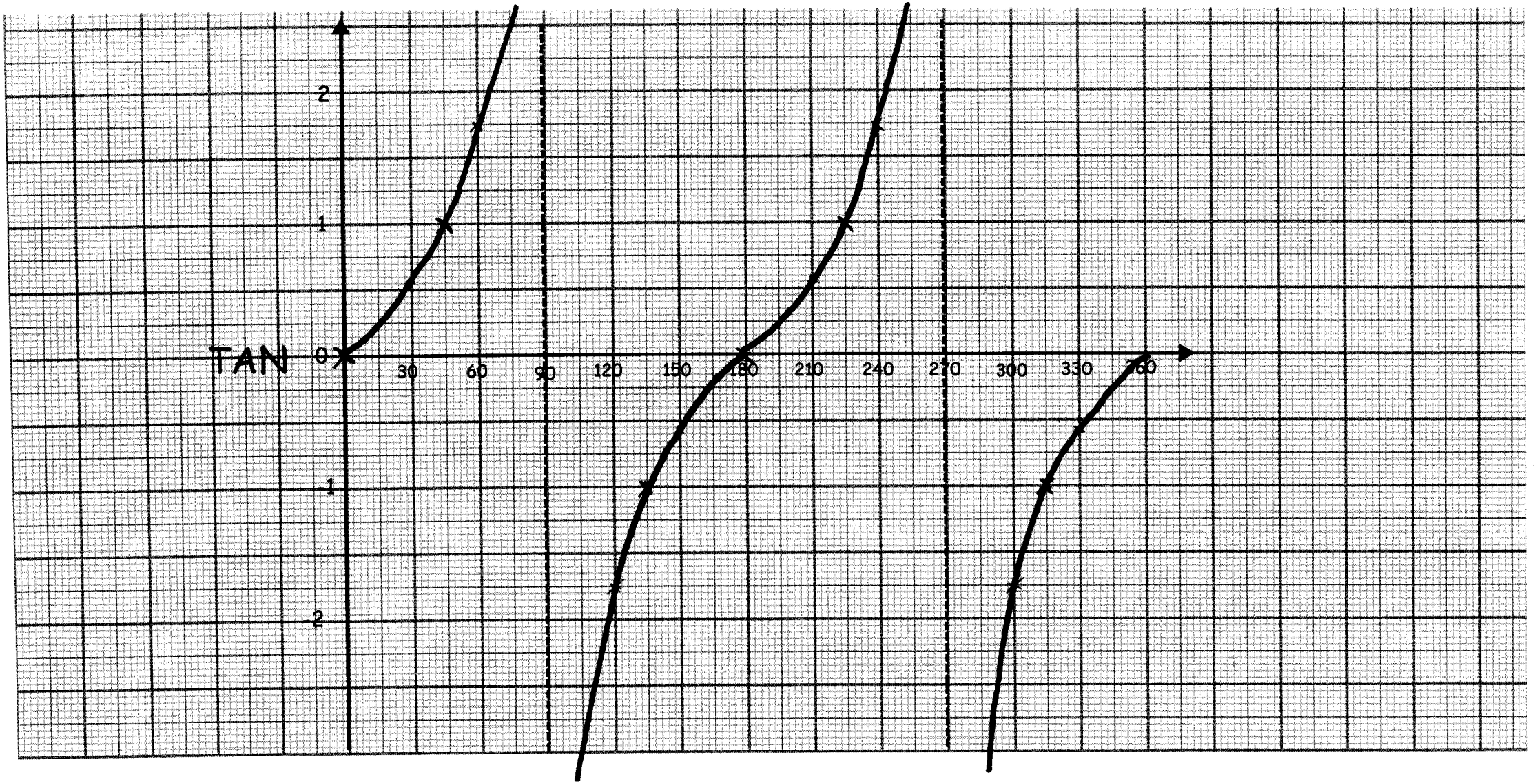
$\sin 30 = \frac{\sqrt{1}}{2} = \frac{1}{2}$ $\cos 45 = \frac{\sqrt{2}}{2}$

$\tan 30 \quad \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

Using this method complete the table

Angle	SIN	COS	TAN
0°	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = \frac{2}{2} = 1$	$\frac{\sqrt{0}}{\sqrt{4}} = 0$
30°	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{\sqrt{2}} = 1$
60°	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = 1/2$	$\frac{\sqrt{3}}{\sqrt{1}} = \sqrt{3}$
90°	$\frac{\sqrt{4}}{2} = \frac{2}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{\sqrt{0}} = \frac{2}{0} = \infty$


SIN
1
0
-1
30
60
90
120
150
180
210
240
270
300
330
360
COS
1
0
-1
30
60
90
120
150
180
210
240
270
300
330
360


TAN
2
1
0
-1
-2
30
60
90
120
150
180
210
240
270
300
330
360