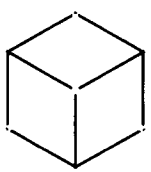
VOLUME and SURFACE AREA

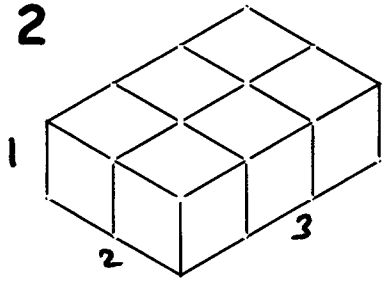
Find the volume of each of these shapes

**1**



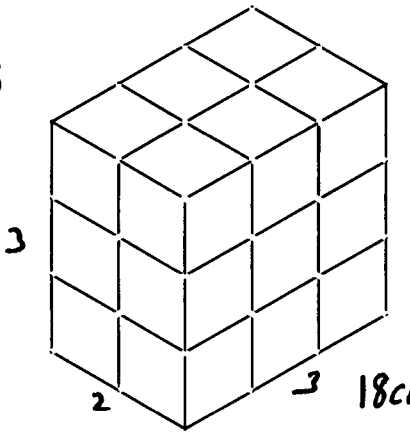
$1cm^3$

**2**



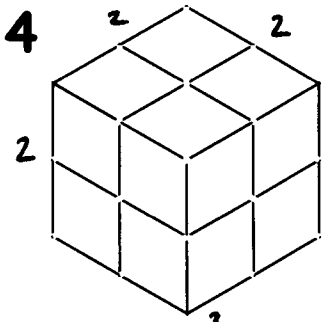

$6cm^3$

**3**



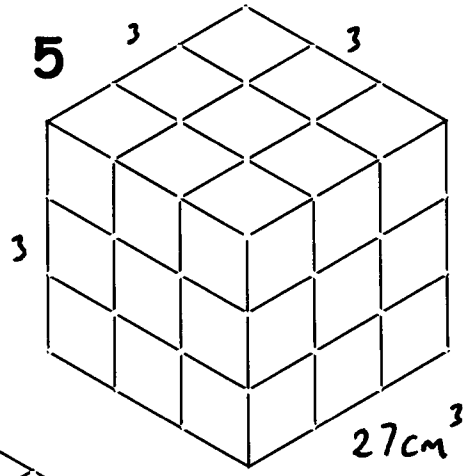

$18cm^3$

**4**



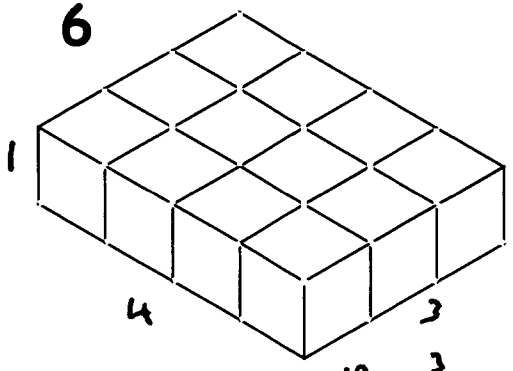

$8cm^3$

**5**



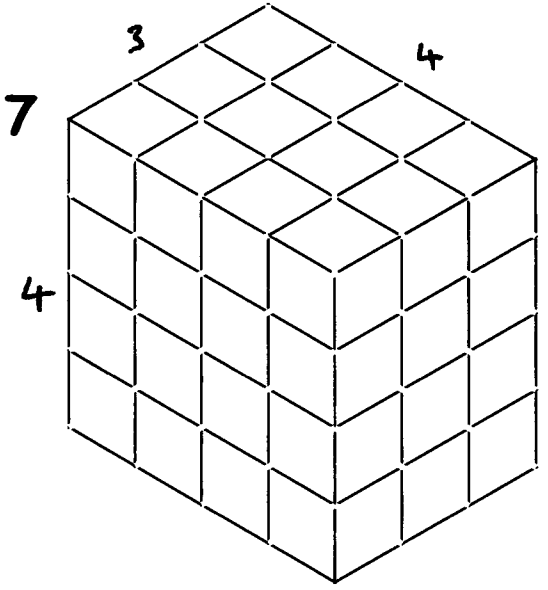

$27cm^3$

**6**



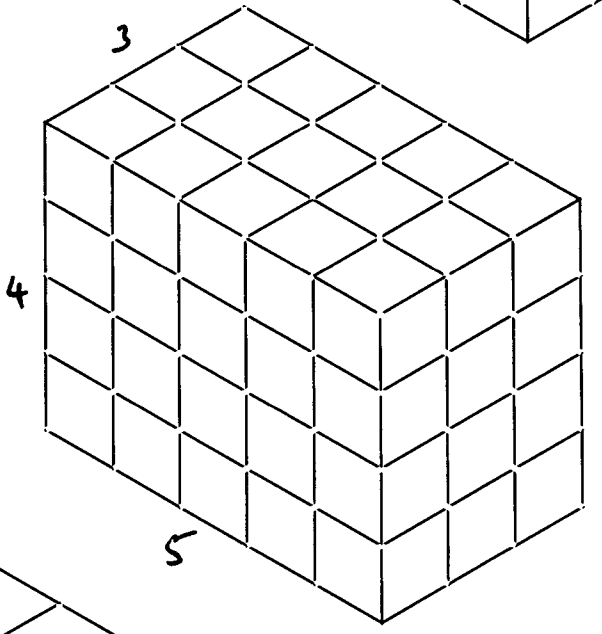

$12cm^3$

**7**



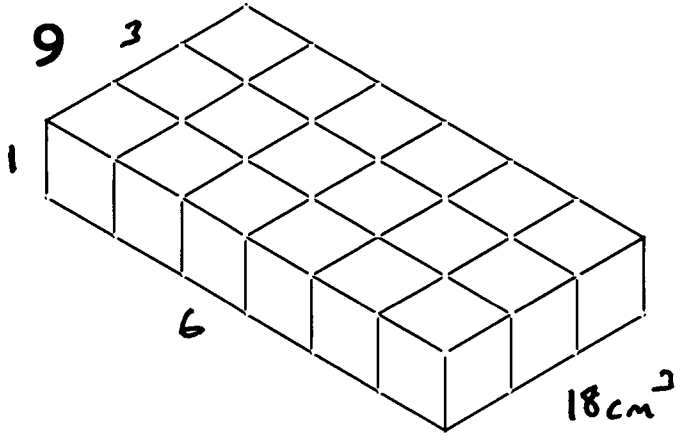

$48cm^3$

**8**



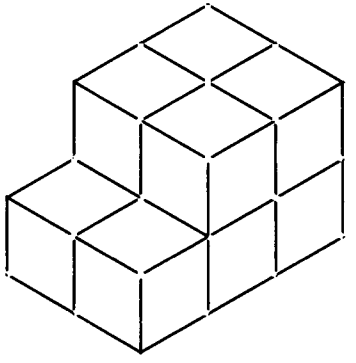

$60cm^3$

**9**



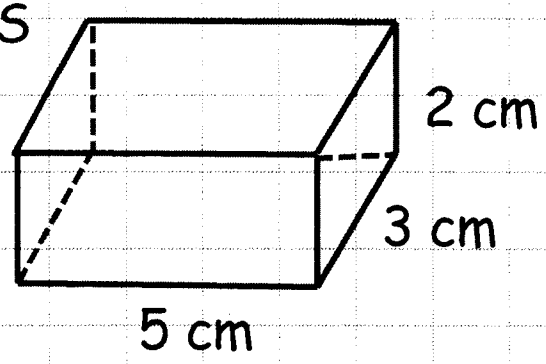

$18cm^3$

**10**



$10cm^3$

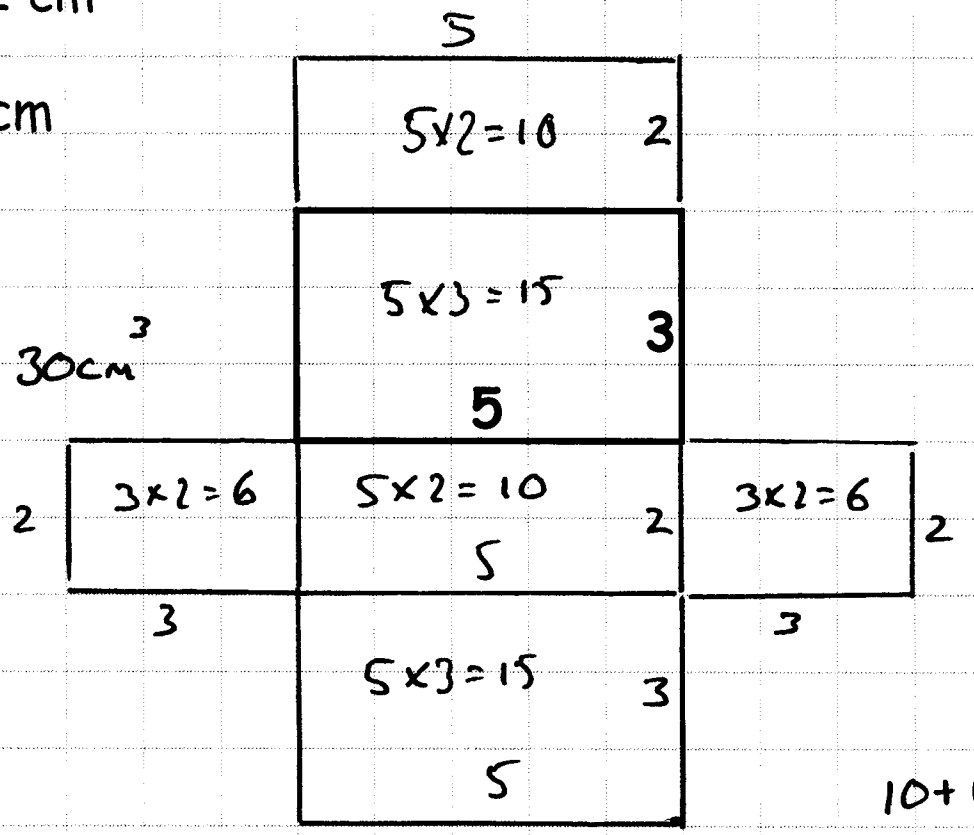
# CUBOIDS




VOLUME = $5 \times 3 \times 2 = 30 cm^3$

How many
Faces = 6
Edges = 12
Corners = 8

Complete the NET of the cuboid

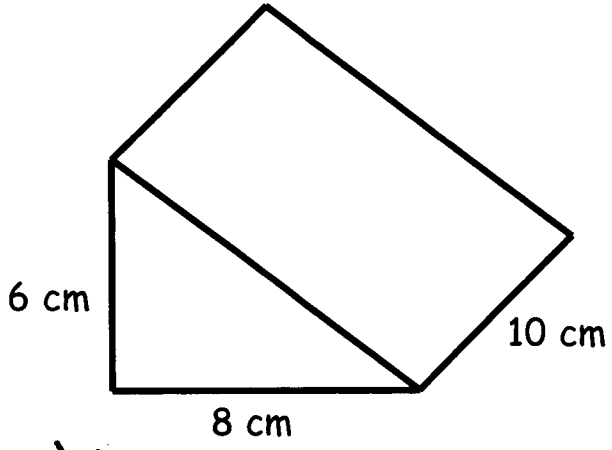



What is the SURFACE AREA of the cuboid?

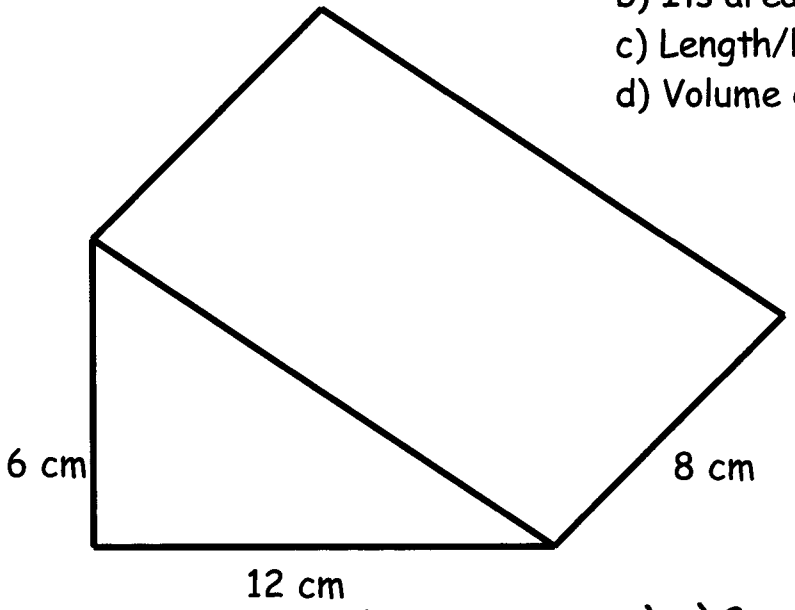
10+15+10+15+6+6
= $62 cm^2$

A prism has the same shape (cross section) running all the way through it
The volume of a prism = Area of cross section × length (or height)

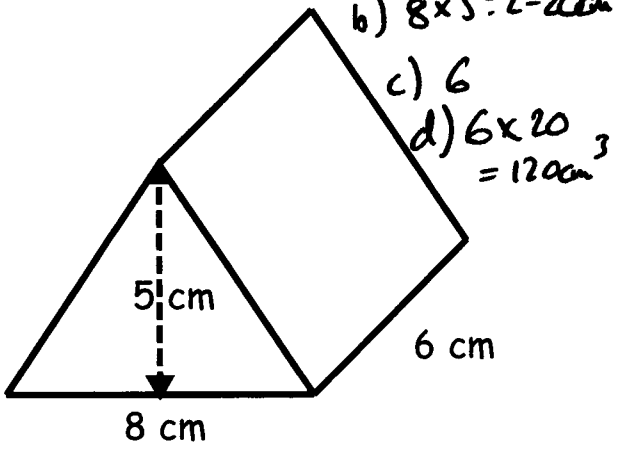
Find the volume of these prisms. For each shape write
a) Name of cross section shape
b) Its area
c) Length/height of prism
d) Volume of prism



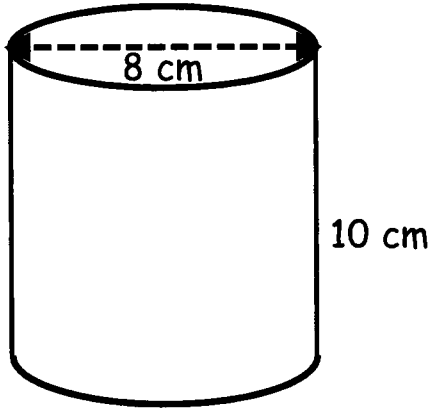

a) triangle
b) $6 \times 8 \div 2 = 24\text{cm}^2$
c) 10 d) $24 \times 10 = 240\text{cm}^3$



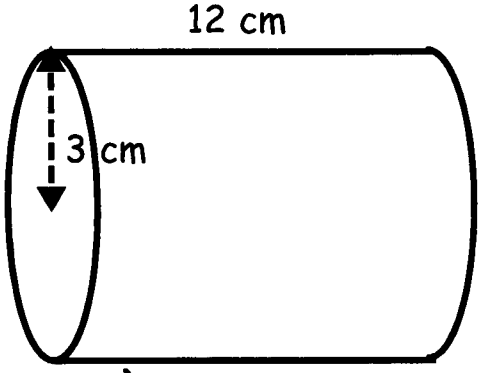

a) triangle b) $6 \times 12 \div 2 = 36\text{cm}^2$ c) 8
d) $36 \times 8 = 288\text{cm}^3$



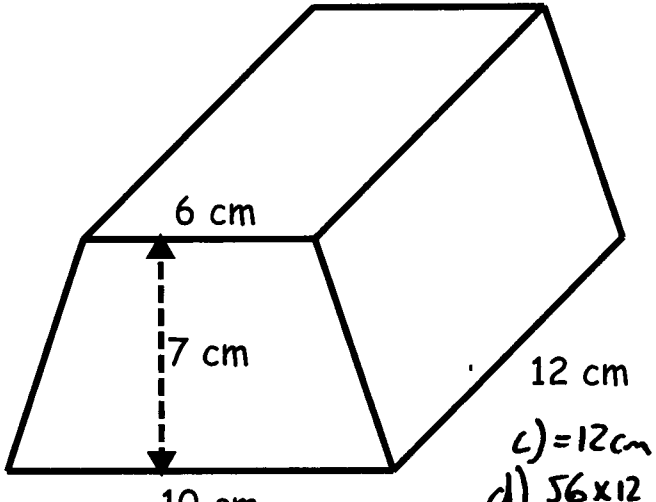

a) triangle
b) $8 \times 5 \div 2 = 20\text{cm}^2$
c) 6
d) $6 \times 20 = 120\text{cm}^3$




a) circle
b) $\pi \times 4^2 = 50.3\text{cm}^2$
c) 10cm d) $10 \times 50.3 = 503\text{cm}^3$



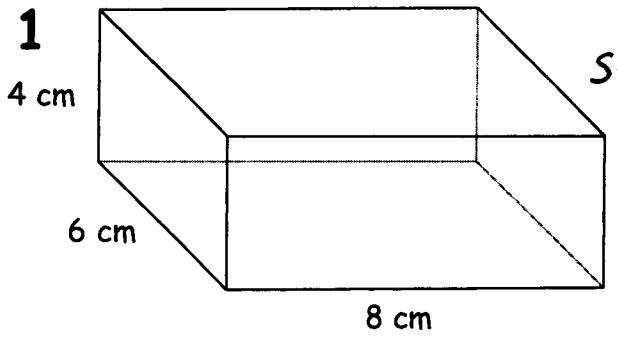

a) circle
b) $\pi \times 3^2 = 28.3\text{cm}^2$
c) 12
d) $28.3 \times 12 = 339.6\text{cm}^3$




a) trapezium
b) $(6+10) \times 7 \div 2 = 56\text{cm}^2$
c) = 12cm
d) $56 \times 12 = 672\text{cm}^3$

$V = 4 \times 6 \times 8 = 192 cm^3$

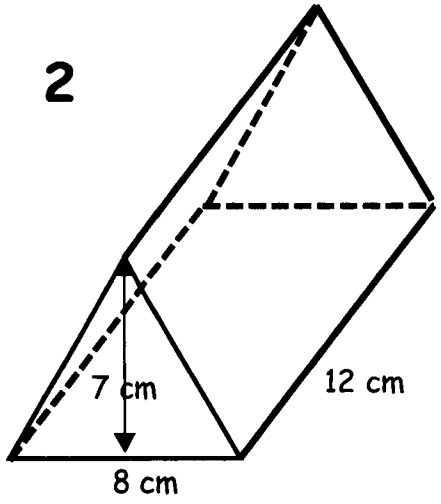
**1**




SA: $4 \times 6 = 24$
$4 \times 8 = 32$
$6 \times 8 = 48$
$104$

$2 \times 104 = 208 cm^2$

**2**



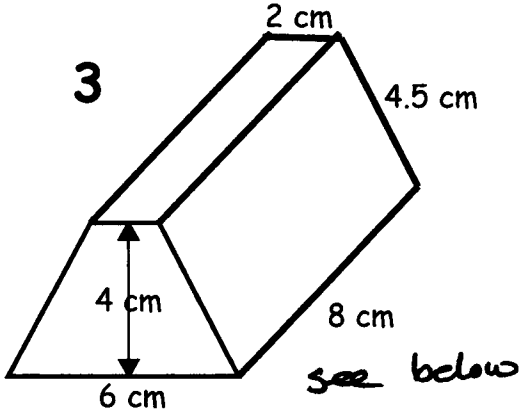

$V = 7 \times 8 \div 2 \times 12 = 336 cm^3$

SA: 2 triangles + 3 rectangles

$= 2 \times 7 \times 8 \div 2 + 3 \times 12 \times 8$

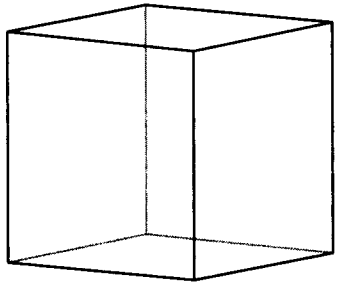
$= 344 cm^2$

**3**




see below

**4**

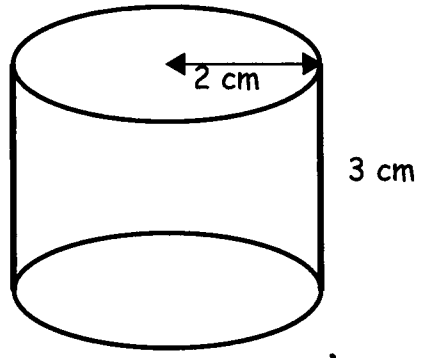


cube side 6 cm

$V = 6 \times 6 \times 6 = 216 cm^3$

$SA = 6 \times 6^2 = 216 cm^2$

**5**




$V = \pi \times 2^2 \times 3$
$= 37.7 cm^3$

SA: 2 circles + 1 rectangle

$= 2 \times \pi \times 2^2 + 3 \times \pi \times 4$

$= 62.8 cm^2$

③ Volume: area of trapezium $\times$ 8

$= (2+6) \times 4 \div 2 \times 8 = 128 cm^3$

Surface area: two trapeziums + top rectangle + base rectangle + 2 side rectangles

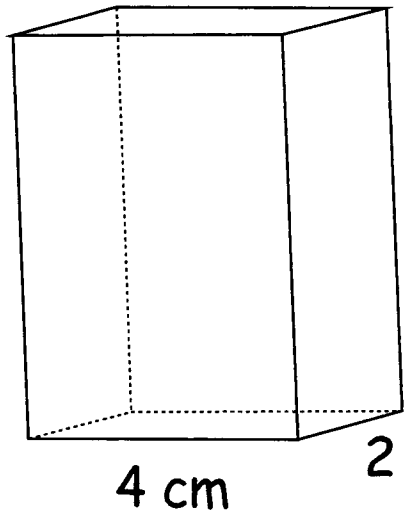
$= 2 \times (2+6) \times 4 \div 2 + 2 \times 8 + 6 \times 8 + 2 \times 8 \times 4.5$

$= 32 + 16 + 48 + 72$

$= 168 cm^2$

Find the volume and surface area of these shapes. Show all working out.

**1**




$V = 4 \times 5 \times 2 = 40 cm^3$

SA = $2 \times 4 = 8$

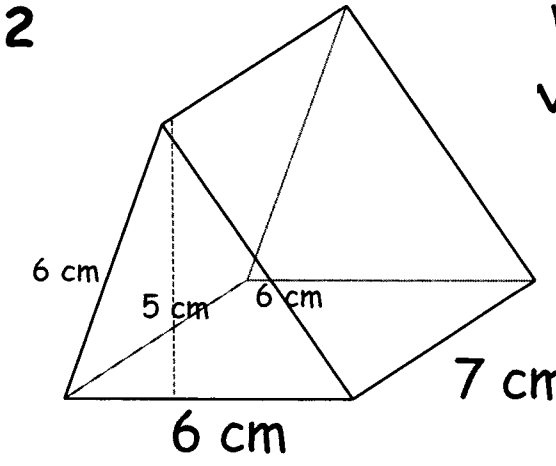
$2 \times 5 = 10$

$4 \times 5 = 20$

38

$38 \times 2 = 76 cm^2$

**2**




$V = 6 \times 5 \div 2 \times 7$

$V = 105 cm^3$

SA = 2 triangles

$2 \times 6 \times 5 \div 2 =$

$2 \times 15 = 30 cm^2$

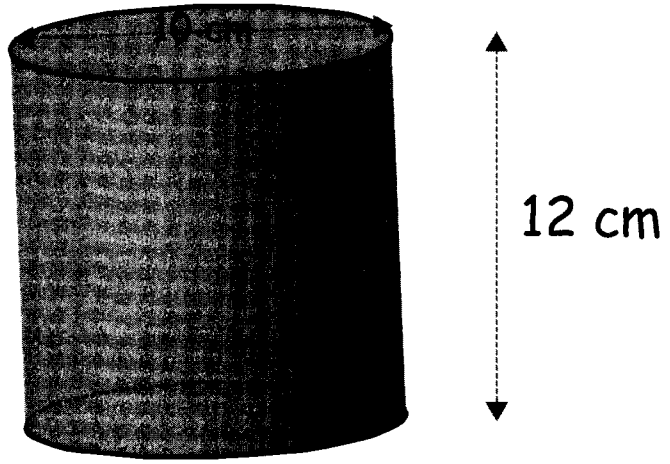
3 rectangles

$3 \times 7 \times 6$

$3 \times 42 = 126 cm^2$

TOTAL = $156 cm^2$

**3**

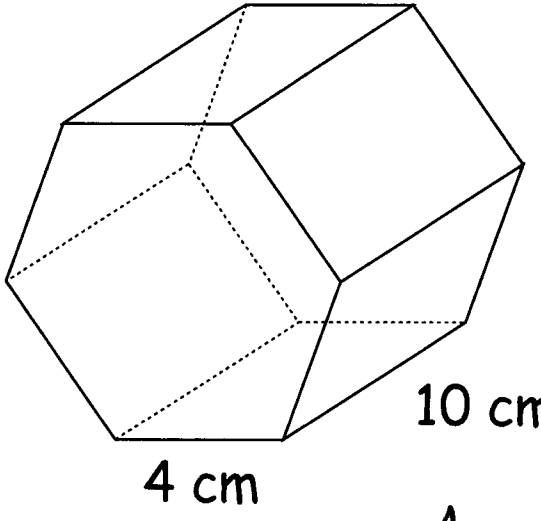



$V = \pi \times 5^2 \times 12 = 942.5 cm^3$

$SA = 2 \times \pi \times 5^2 + \pi \times 10 \times 12 = 534.1 cm^2$

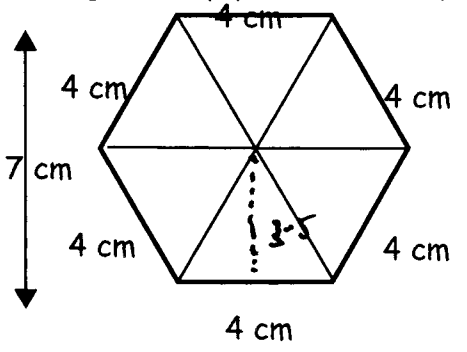
2 x circles + rectangle

**4**




Hint

The cross section is a regular hexagon. Split it into 6 triangles, find the area of one triangle multiply the answer by 6




Area of hexagon

6 triangles $6 \times 4 \times 3.5 \div 2 = 42 cm^2$

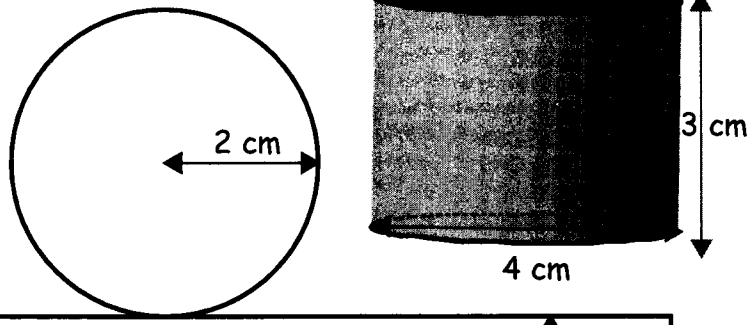
Volume = $42 \times 10 = 420 cm^3$

SA = $2 \times 42 + 6 \times 4 \times 10 = 324 cm^2$

2 hexagons + 6 rectangles

## 1

Here is a picture of a cylinder and its net. Find its volume and surface area



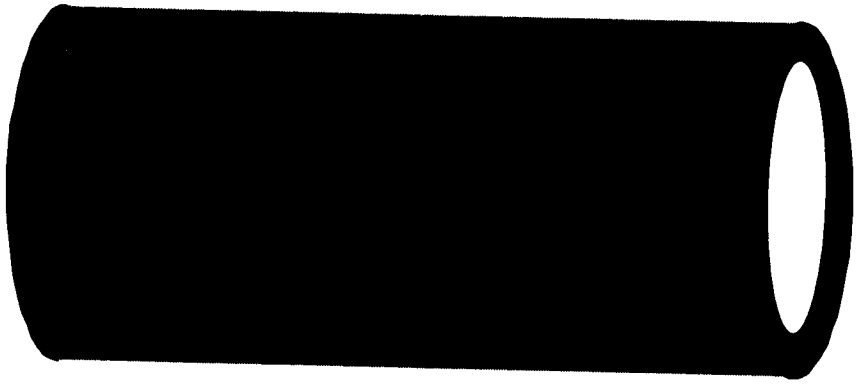

$V = \pi \times 2^2 \times 3 = 37.7 cm^3$

$S.A = 2 \times \pi \times 2^2 + 3 \times \pi \times 4 = 62.8 cm^2$

3 cm

base

## 4



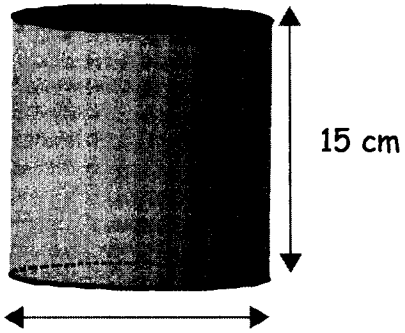
This is a length of concrete pipe. The pipe is 5 m long. The radius of the pipe is 0.4 m. The radius of the hole is 0.3 m. Calcualte the volume of the pipe.

$V = \pi \times 0.4^2 \times 5 - \pi \times 0.3^2 \times 5$

$V = 2.5 - 1.4 = 1.1 m^3$

Find the volume and surface area for questions 2 and 3

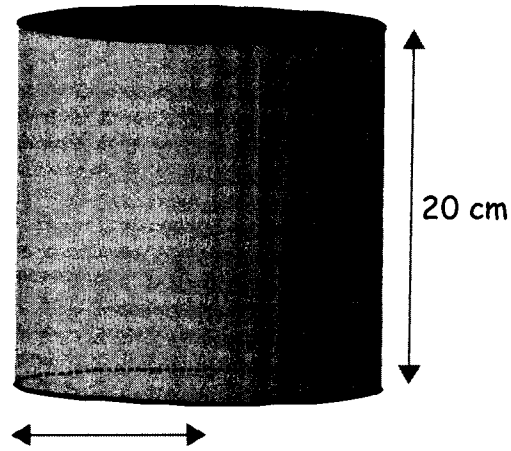
## 2




$V = \pi \times 5^2 \times 15 = 1178.1 cm^3$

$SA = 2 \times \pi \times 5^2 + 15 \times \pi \times 10 = 628.3 cm^2$

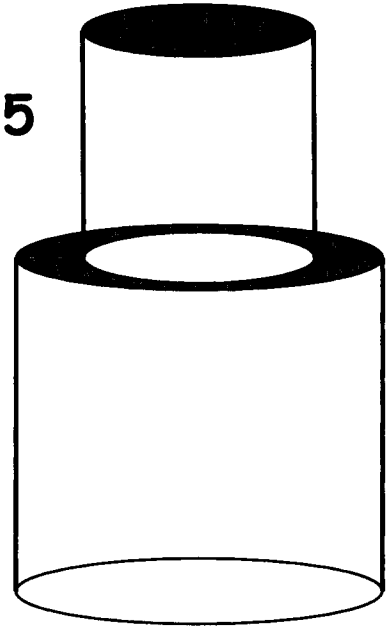
## 3




$V = \pi \times 8^2 \times 20 = 4021.2 cm^3$

$SA = 2 \times \pi \times 8^2 + 20 \times \pi \times 16 = 1407.4 cm^2$

## 5



Two cylinders are placed on top of each other. The top cylinder has a radius of 5 cm and a height of 12 cm. The bottom cylinder has a diameter of 16 cm and a height of 20 cm.

Calculate the volume of this object and its surface area.

$V = \pi \times 5^2 \times 12 + \pi \times 8^2 \times 20$

$V = 942.5 + 4021.2 = 4963.7 cm^3$

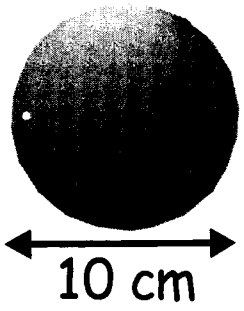
$SA = \pi \times 5^2 + 12 \times \pi \times 10 + \pi \times 8^2 - \pi \times 5^2 + 20 \times \pi \times 16 + \pi \times 8^2$

$= 1784.4 cm^2$

Volume of a sphere is $\frac{4}{3}\pi r^3$ this means $4 \times \pi \times r \times r \times r \div 3$

The surface area of a sphere is $4 \pi r^2$ Find the volume and surface area of these shapes to 1 d.p. Don't forget the units.

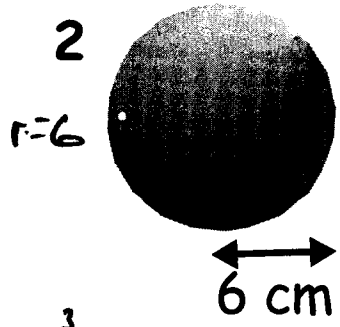
**1**




$V = \frac{4}{3} \times \pi \times 5^3 = 523.6 cm^3$

$SA = 4 \times \pi \times 5^2$
$= 314.2 cm^2$

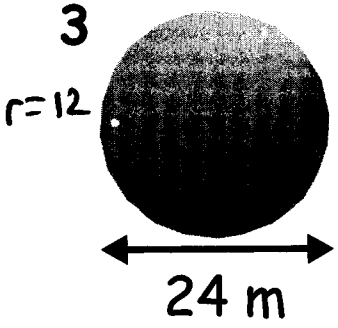
**2**




$V = 904.8 cm^3$

$SA = 452.4 cm^2$

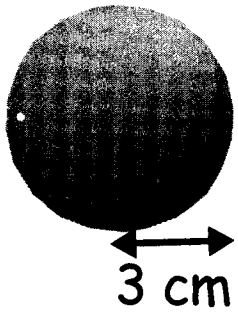
**3**




$V = 7238.2 m^3$

$SA = 1809.6 m^2$

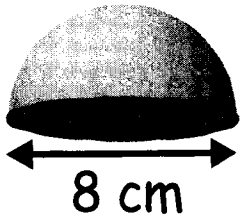
**4**




$V = 113.1 cm^3$

$SA = 113.1 cm^2$

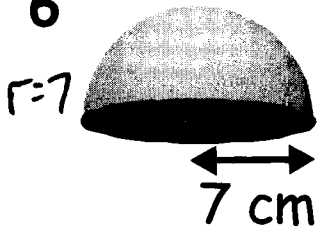
**5**




$V = \frac{4}{3} \times \pi \times 4^3 \div 2 = 134.0 cm^3$

$SA = 4 \times \pi \times 4^2 \div 2$
$+ \pi \times 4^2$
$= 150.8 cm^2$

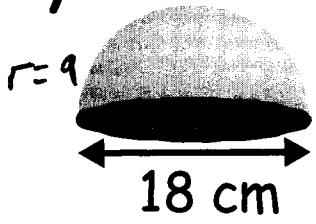
**6**




$V = 718.4 cm^3$

$SA = 461.8 cm^2$

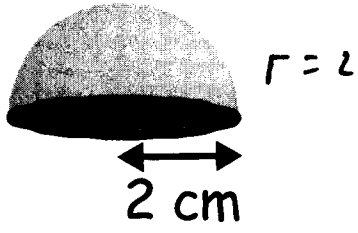
**7**




$V = 1526.8 cm^3$

$SA = 763.4 cm^2$

**8**




$V = 16.8 cm^3$

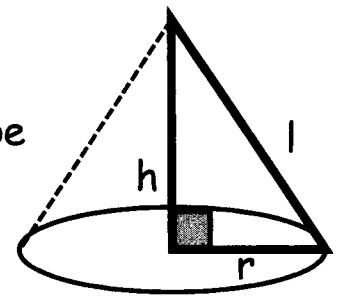
$SA = 37.7 cm^2$

In general the Volume of a pyramid is $\frac{1}{3}$ x base area x perpendicular height

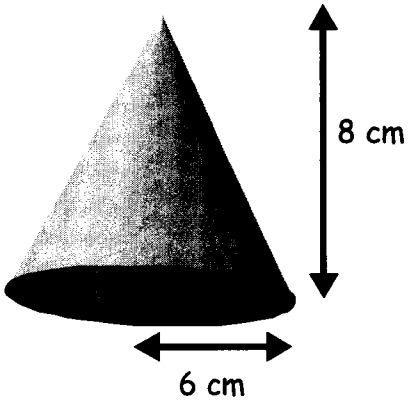
For a cone $V = \frac{1}{3}\pi r^2 h$ Surface Area = $\pi r^2$ (base) + $\pi rl$ (curved surface)

l is the slant height and can be found using pythagoras.

$l^2 = r^2 + h^2$




Find the volume and surface area of these pyramids




$r = 6$

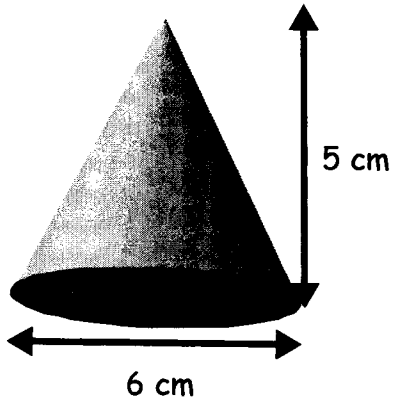
$h = 8$

$l = \sqrt{(6^2+8^2)} = \sqrt{100} = 10$

$V = \frac{1}{3} \times \pi \times 6^2 \times 8 = 301.6 cm^3$

S.A. $= \pi \times 6^2 + \pi \times 6 \times 10$

$= 301.6 cm^2$




$r = 3$

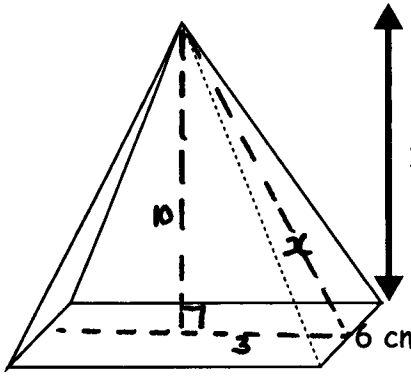
$h = 5$

$l = \sqrt{3^2+5^2} = 5.8$

$V = 47.1 cm^3$

S.A. $= \pi \times 3^2 + \pi \times 3 \times 5.8$

$= 82.9 cm^2$




$V = \frac{1}{3} \times 6^2 \times 10$

$= 120 cm^3$

$x = \sqrt{10^2+3^2} = 10.4$

S.A =

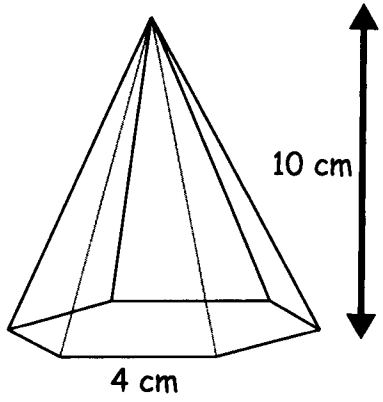
base $6 \times 6 = 36 cm^2$

4 triangles $4 \times 6 \times 10.4 \div 2$

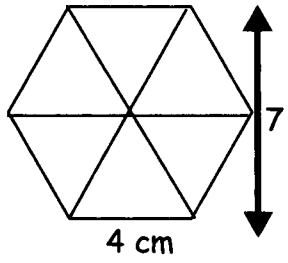
$124.8 cm^2$

TOTAL $= 160.8 cm^2$

$\sqrt{10^2+3.5^2} = 10.6$




base




Area of 1 small triangle

$4 \times 3.5 \div 2 = 7 cm^2$

Area of hexagon

$7 \times 6 = 42 cm^2$

$V = \frac{1}{3} \times 42 \times 10$

$V = 140 cm^3$

SA = 1 hexagon + 6 triangles

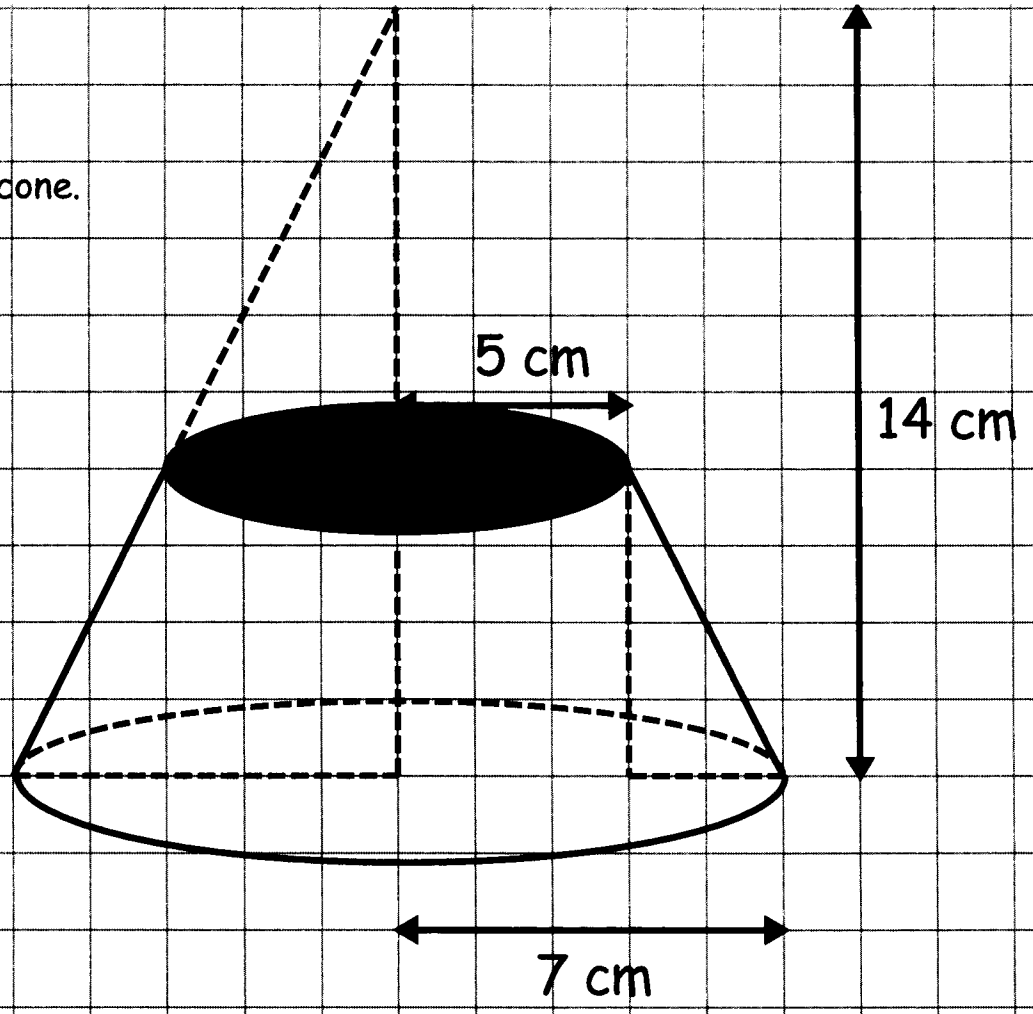
$= 42 + 6 \times 10.6 \times 4 \div 2 = 169.2 cm^2$

# Volume of frustrum

A frustrum is formed when the top is sliced off a cone.

Volume of a cone = $\frac{1}{3}\pi r^2 h$

You may need to use trigonometry or similar triangles to work out missing dimensions.




Volume of big cone = $\frac{1}{3} \times \pi \times 7^2 \times 14$

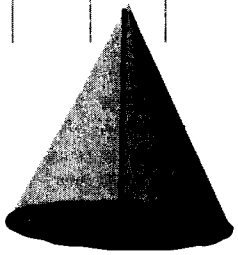
$= 718.4\,cm^3$

Volume of small cone = $\frac{1}{3} \times \pi \times 5^2 \times 10$

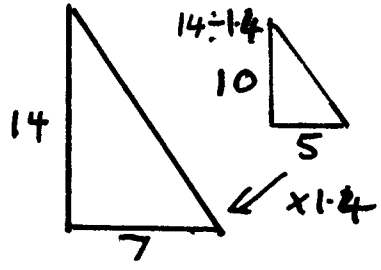
$= 261.8\,cm^3$

Volume of Frustrum = $718.4 - 261.8$

$= 456.6\,cm^3$

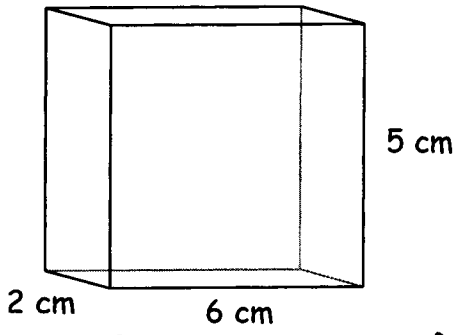


Similar triangles to find the height of the small cone



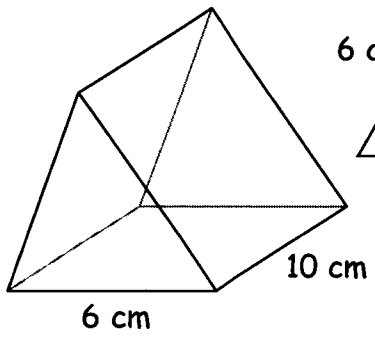
Calculate the volume and surface area of these shapes

1) A Cuboid

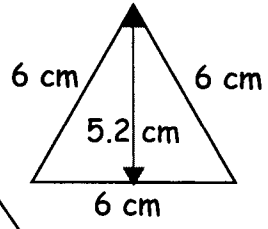



$V = 60\,cm^3$ $SA = 104\,cm^2$

2) A triangular prism



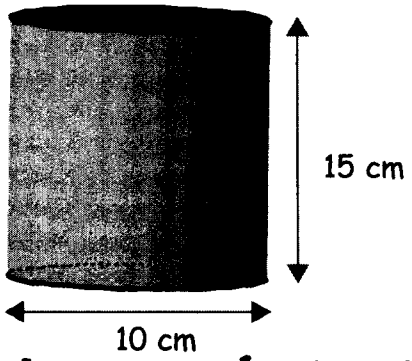

cross-section




$V = 156\,cm^3$

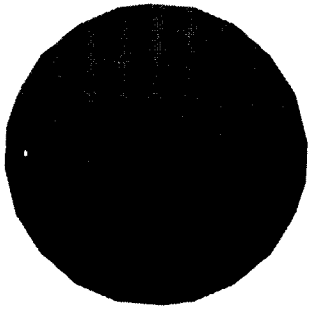
$SA = 211.2\,cm^2$

3) A Cylinder




$V = 1178.1\,cm^3$ $SA = 628.3\,cm^2$

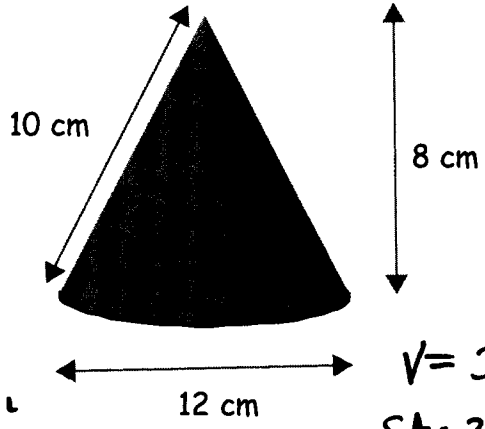
4) A sphere



$V = 268.1\,cm^3$

$SA = 201.1\,cm^2$

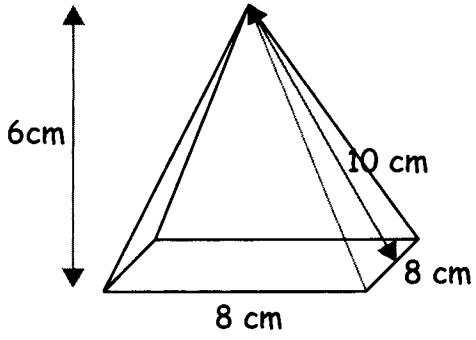
5) A cone




$V = 301.6\,cm^3$

$SA = 301.6\,cm^2$

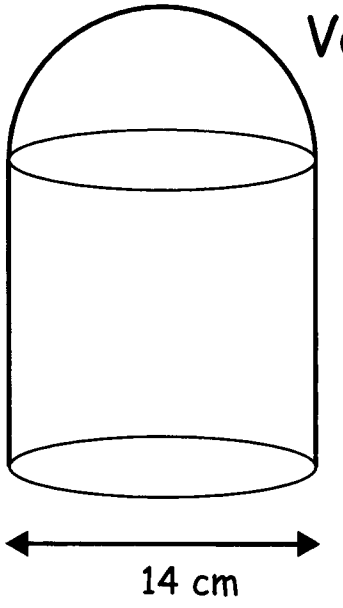
6) A square based pyramid




$V = 128\,cm^3$

$SA = 224\,cm^2$

## 1 Volume and surface area




V = cylinder + hemisphere

V = 2463·0 + 718·4

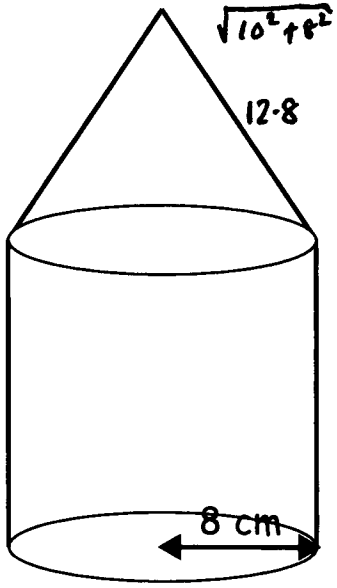
V = 3181·4 cm³

SA = curved surfaces of hemisphere + cylinder + circle base

$SA = \frac{4\times\pi\times7^2}{2} + 16\times\pi\times14 + \pi\times7^2$

SA = 1165·5 cm²

## 2 Volume and surface area




V = cone + cylinder

V = 670·2 + 3015·9

= 3686·1 cm³

Surface Area

$= \pi\times8\times12\cdot8$

+

$= 15\times\pi\times16$

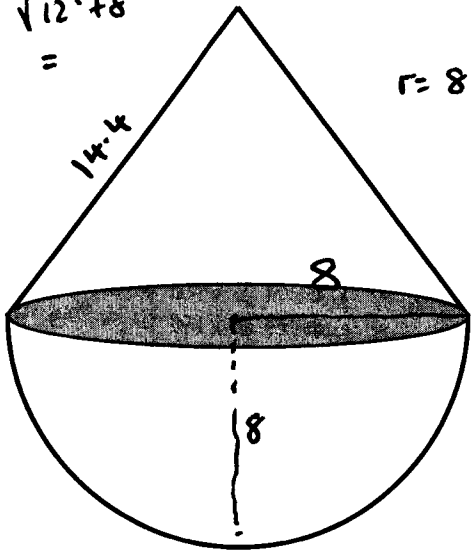
+

$\pi\times8^2$

= 1276·7 cm²

## 3 Volume and Surface Area

$\sqrt{12^2+8^2}$ = 14·4




Volume

hemisphere = 1072·3 cm³

cone = 804·2 cm³

TOTAL = 1876·5 cm³

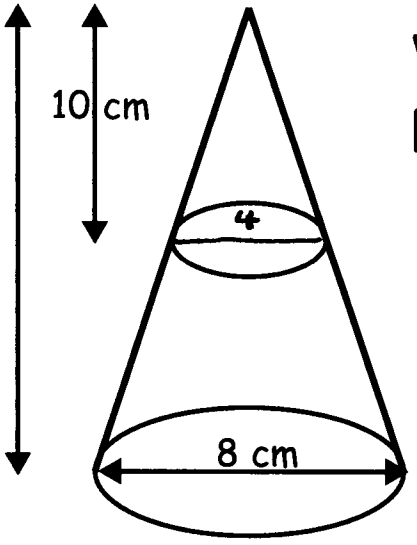
Surface area.

$\pi\times8\times14\cdot4 + 4\times\pi\times8^2 \div 2$

= 764·0 cm²

## 4

The top of a cone is removed to make a frustrum. Find the volume of the frustrum.




Volume of Frustrum

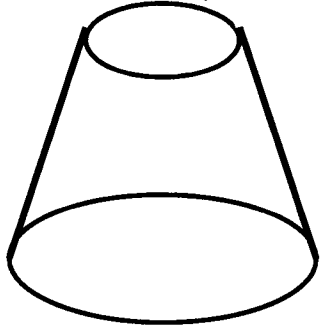
$\frac{1}{3}\times\pi\times4^2\times20$

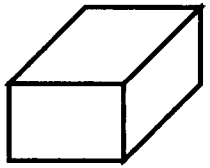
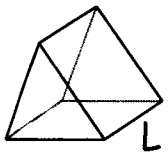
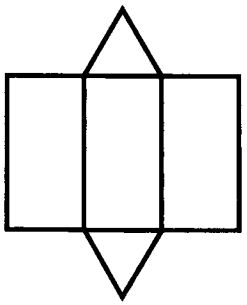
−

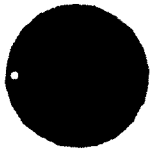
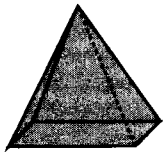
$\frac{1}{3}\times\pi\times2^2\times10$

= 335·1 − 41·9

= 293·2 cm³

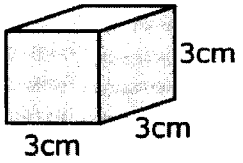


| NAME | VOLUME OF PRISMS<br>area of cross section x length | SURFACE AREA |
|---|---|---|
| Cube/ Cuboid |  <br>length x width x height | add areas of all the 6 faces together |
| Cylinder |  $V = \pi r^2 h$ | $\pi r^2$<br>$\pi d \times h$<br>$\pi r^2$ |
| Triangular Prism |  <br>L<br>V = base x height ÷ 2 x Length |  <br>3 rectangles + 2 triangles |
| Any prism | V = area of cross section x height | Area of all the faces added together |

| NAME | VOLUME | SURFACE AREA |
|---|---|---|
| Sphere |  $\frac{4}{3} \pi r^3$ | $4 \pi r^2$ |
| Hemisphere |  <br>$\frac{4}{3} \pi r^3 \div 2$ | $4 \pi r^2 \div 2 + \pi r^2$ |
| Pyramid |  <br>$\frac{1}{3}$ x area of base x height | Area of all the faces added together<br>May need pythagoras |
| Cone |  $\frac{1}{3} \pi r^2 h$ | $\pi r^2 + \pi r L$<br>L is the slant height<br>$L^2 = r^2 + h^2$ |

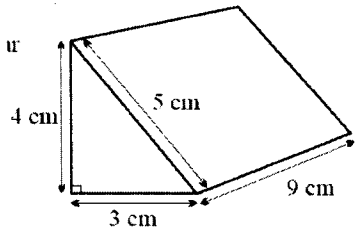
| Shape | Cube/Cuboid | Prism | Pyramid | Sphere |
|---|---|---|---|---|
| | V =length x width x height | V = area of cross section x height | V =area of base x height ÷ 3 | $V = \frac{4}{3}\pi r^3 \quad S.A = 4\pi r^2$ |

Find the Volume and Surface Area of each shape.




$V = 3^3 = 27\,cm^3$

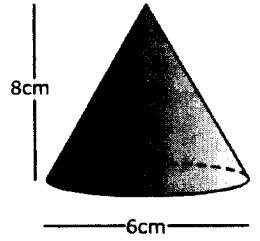
$SA = 6 \times 9 = 54\,cm^2$




$V = 4 \times 3 \div 2 \times 9$
$= 54\,cm^3$

$SA = 120\,cm^2$

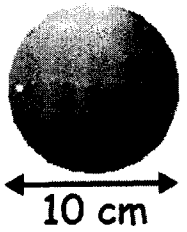
$\sqrt{8^2+3^2}$
$= 8.5$




$S.A. = \pi r^2 + \pi rl$

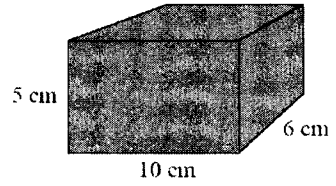
$V = 75.4\,cm^3$

$SA = \pi \times 3^2 + \pi \times 3 \times 8.5$
$= 108.4cm$



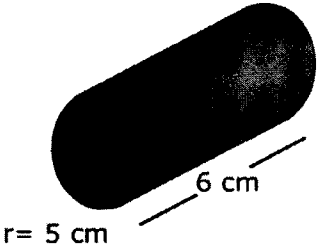

$V = 523.6\,cm^3$

$SA = 314.2cm^2$



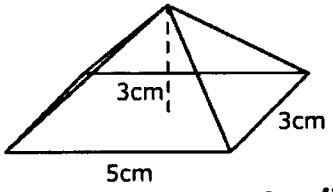

$V = 5 \times 10 \times 6 = 300cm^3$

$SA = 280\,cm^2$




$V = 471.2\,cm^3$

$SA = 345.6\,cm^2$

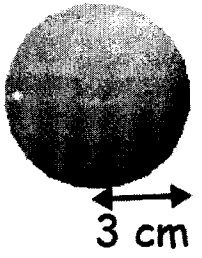



$V = \frac{1}{3} \times 5 \times 3 \times 3 = 15cm^3$

$SA = 5 \times 3 + \frac{2 \times 5 \times 3.4}{2} + \frac{2 \times 3 \times 3.9}{2}$
$= 43.7cm^2$

$\sqrt{3^2+2.5^2} = 3.9$

$\sqrt{3^2+1.5^2} = 3.4$




$V = 113.1\,cm^3$

$SA = 113.1cm^2$

AREAS Triangle = base x height ÷ 2 Trapezium = add parallel sides x height ÷ 2 Circle = $\pi$ x radius$^2$