SURDS

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Numbers and their square roots that can be SIMPLIFIED

1	2 ✖	3 ✖	4	5 ✖	6 ✖	7 ✖	8 2√2	9	10 ✖
11 ✖	12	13 ✖	14 ✖	15 ✖	16	17 ✖	18	19 ✖	20
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61 ✖	62 ✖	63	64	65 ✖	66 ✖	67 ✖	68	69 ✖	70 ✖
71 ✖	72	73 ✖	74 ✖	75	76	77 ✖	78 ✖	79 ✖	80
81	82 ✖	83 ✖	84	85 ✖	86 ✖	87 ✖	88	89 ✖	90
91 ✖	92	93 ✖	94 ✖	95 ✖	96	97 ✖	98	99	100

Numbers from 1 to 100 that have a SQUARE NUMBER as a factor. Starting with the highest square numbers going to the lowest. (49, 36, 25, 16, 9, 4)

49 98 = 49 x 2 **36** 72 = 36 x 2 **25** 50 = 25 x 2 75 = 25 x 3 **100 = 25 x 4** *

16 32 = 16 x 2 48 = 16 x 3 **64 = 16 x 4** * 80 = 16 x 5 96 = 16 x 6

9 18 = 9 x 2 27 = 9 x 3 **36 = 9 x 4** * 45 = 9 x 5 54 = 9 x 6 63 = 9 x 7 **72 = 9 x 8** *

81 = 9 x 9 * 90 = 9 x 10 99 = 9 x 11

4 8 = 4 x 2 12 = 4 x 3 **16 = 4 x 4** * 20 = 4 x 5 24 = 4 x 6 28 = 4 x 7 **32 = 4 x 8** *

36 = 4 x 9 * 40 = 4 x 10 44 = 4 x 11 **48 = 4 x 12** * 52 = 4 x 13 56 = 4 x 14 60 = 4 x 15

64 = 4 x 16 * 68 = 4 x 17 **72 = 4 x 18** * 76 = 4 x 19 **80 = 4 x 20** * 84 = 4 x 21 88 = 4 x 22

92 = 4 x 23 **96 = 4 x 24** * **100 = 4 x 25** * *** means already done**

Surds - the first few square numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 .....

	Find the largest square number factor		
$\sqrt{32}$	$\sqrt{16 \times 2}$	$\sqrt{16} \times \sqrt{2}$	$4\sqrt{2}$
$\sqrt{8}$			
$\sqrt{50}$			
$\sqrt{27}$			
$\sqrt{48}$			
$\sqrt{98}$			
$\sqrt{12}$			
$\sqrt{20}$			
$\sqrt{18}$			
$\sqrt{24}$			
$\sqrt{75}$			

Simplify

1) $\sqrt{2} \times \sqrt{8}$

2) $4 \times \sqrt{8}$

3) $\sqrt{3} \times \sqrt{12}$

4) $(\sqrt{2})^2$

5) $3\sqrt{3} \times \sqrt{12}$

6) $\frac{\sqrt{12}}{\sqrt{3}}$

7) $\frac{\sqrt{54}}{\sqrt{2}}$

8) $(\sqrt{3})^4$

9) $3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2}$

10) $\sqrt{2}(\sqrt{2} - 3)$

Rules for Surds (Writing numbers using square roots)

Addition and Subtraction

1) $\sqrt{2} + \sqrt{2} =$

2) $\sqrt{5} + \sqrt{5} + \sqrt{5} =$

3) $\sqrt{2} + 3\sqrt{2} =$

4) $2\sqrt{3} + 5\sqrt{3} =$

5) $\sqrt{2} + \sqrt{3} =$

6) $4\sqrt{3} - 2\sqrt{3} =$

7) $\sqrt{5} - \sqrt{2} =$

8) $4\sqrt{3} + 2\sqrt{3} - \sqrt{3} + \sqrt{5} =$

Multiplication

7) $\sqrt{3} \times \sqrt{5} =$

8) $\sqrt{2} \times \sqrt{7} =$

9) $\sqrt{7} \times \sqrt{6} =$

10) $3 \times \sqrt{5} =$

11) $4 \times \sqrt{3} =$

12) $\sqrt{3} \times \sqrt{12} =$

Division

13) $\frac{\sqrt{8}}{\sqrt{4}} =$

14) $\frac{\sqrt{18}}{\sqrt{6}} =$

15) $\frac{\sqrt{15}}{3} =$

16) $\frac{\sqrt{18}}{6} =$

17) $\frac{\sqrt{18}}{\sqrt{2}} =$

Surds, the Key Skills

1 Finding the highest SQUARE NUMBER FACTOR.

The first few square numbers are 4, 9, 16, 25, 36, 49, 64, 81, 100

$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{32} =$

2 ADDING AND TAKING LIKE TERMS

$a)\ 3\sqrt{2} + 4\sqrt{2} =$ $b)\ 5\sqrt{3} - \sqrt{3} =$

$c)\ 6\sqrt{2} + 3\sqrt{2} =$ $d)\ 5\sqrt{7} + 2\sqrt{7} =$

$e)\ 6\sqrt{5} - 4\sqrt{5} =$ $f)\ 3\sqrt{2} + 7\sqrt{2} - 6\sqrt{2} =$

Making them the same

$g)\ \sqrt{8} + \sqrt{18} =$ $h)\ \sqrt{12} + \sqrt{48} =$

$i)\ 3\sqrt{8} + \sqrt{50} =$ $j)\ \sqrt{90} - \sqrt{40} =$

$k)\ 3\sqrt{96} - 2\sqrt{24} =$ $l)\ 5\sqrt{63} + 2\sqrt{28} =$

3 TIMES and DIVIDES

$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$ $2 \times \sqrt{3} \times \sqrt{6} = 2 \times \sqrt{3 \times 6} = 2 \times \sqrt{18}$

$$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

$a)\ \sqrt{5} \times \sqrt{10} =$ $b)\ \sqrt{2} \times \sqrt{32} =$

$c)\ \sqrt{6} \times \sqrt{10} =$ $d)\ \sqrt{3} \times \sqrt{3} =$

$e)\ \frac{\sqrt{20}}{\sqrt{10}} =$ $f)\ \frac{\sqrt{80}}{\sqrt{20}} =$ $g)\ \frac{\sqrt{27}}{\sqrt{3}} =$ $h)\ \frac{\sqrt{50}}{\sqrt{2}} =$

4 MULTIPLYING BRACKETS

Single bracket $\sqrt{2}(1+\sqrt{2}) = \sqrt{2}\times 1 + \sqrt{2}\times\sqrt{2}$

$a)\ \sqrt{3}(2+\sqrt{3}) =$

$b)\ \sqrt{6}(\sqrt{6}-4) =$

$c)\ \sqrt{2}(5+2\sqrt{2}) =$

$d)\ \sqrt{5}(2\sqrt{5}+\sqrt{3}) =$

Two brackets $(\sqrt{2}+1)(3-\sqrt{2}) = \sqrt{2}\times 3 - \sqrt{2}\times\sqrt{2} + 1\times 3 - 1\times\sqrt{2}$

$a)\ (\sqrt{3}+1)(3+\sqrt{3}) =$

$b)\ (2\sqrt{5}+1)(4+\sqrt{5}) =$

$c)\ (\sqrt{2}-3)(\sqrt{2}-1) =$

REMEMBER

TRUE $\sqrt{2}\times\sqrt{4} = \sqrt{2\times 4} = \sqrt{8}$ $\frac{\sqrt{4}}{\sqrt{2}} = \sqrt{\frac{4}{2}} = \sqrt{2}$

FALSE $\sqrt{2}+\sqrt{4}\ is\ not\ \sqrt{2+4}$ $\sqrt{2}-\sqrt{4}\ is\ not\ \sqrt{2-4}$

Rationalising the denominator

When we rationalise the denominator of a fraction we remove the square root from the denominator, they can still be present in the numerator.

There are two situations

1) The denominator contains a single term. For example $\frac{2}{\sqrt{3}}$ or $\frac{5}{2\sqrt{3}}$ or $\frac{2+\sqrt{5}}{\sqrt{3}}$

In each of these cases multiply the original fraction by a new fraction which has the denominator of the original fraction as its numerator and denominator.

$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ then multiply them out and simplify $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$$\frac{5}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{10\sqrt{3}}{4\times3} = \frac{5\sqrt{3}}{6}$$

$$\frac{2+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}(2+\sqrt{5})}{3} = \frac{2\sqrt{3}+\sqrt{15}}{3}$$

2) The denominator contains two terms. For example $\frac{2}{1+\sqrt{3}}$ or $\frac{5}{2\sqrt{3}-1}$

In each of these cases multiply the original fraction by a new fraction which has the denominator of the original fraction WITH ITS SIGN CHANGED as its numerator and denominator. This idea is based on the difference of two squares. $(a-b)(a+b) = a^2 - b^2$ $\quad (1-\sqrt{3})(1+\sqrt{3}) = 1^2 - (\sqrt{3})^2 = 1 - 3 = -2$

$\frac{2}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$ then multiply them out and simplify $\frac{2\times1-2\sqrt{3}}{1\times1-1\times\sqrt{3}+\sqrt{3}\times1-\sqrt{3}\times\sqrt{3}}$

This simplifies to make $\frac{2-2\sqrt{3}}{1-3} = \frac{2-2\sqrt{3}}{-2} = \sqrt{3}-1$

$$\frac{5}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1} = \frac{10\sqrt{3}+5}{2\sqrt{3}\times2\sqrt{3}+2\sqrt{3}\times1-1\times2\sqrt{3}-1\times1} = \frac{10\sqrt{3}+5}{11}$$

Rationalise the denominator of these fractions

1) $\frac{3}{\sqrt{5}}$ 2) $\frac{7}{\sqrt{2}}$ 3) $\frac{5}{4\sqrt{6}}$ 4) $\frac{5}{4+\sqrt{6}}$ 5) $\frac{5}{\sqrt{3}-2}$ 6) $\frac{\sqrt{2}-1}{\sqrt{3}+3}$