SURDS

1 1	2 ✖	3 ✖	4 2	5 ✖	6 ✖	7 ✖	8 $2\sqrt{2}$	9 3	10 ✖
11 ✖	12 $2\sqrt{3}$	13 ✖	14 ✖	15 ✖	16 4	17 ✖	18 $3\sqrt{2}$	19 ✖	20 $2\sqrt{5}$
21 ✖	22 ✖	23 ✖	24 $2\sqrt{6}$	25 5	26 ✖	27 $3\sqrt{3}$	28 $2\sqrt{7}$	29 ✖	30 ✖
31 ✖	32 $4\sqrt{2}$	33 ✖	34 ✖	35 ✖	36 6	37 ✖	38 ✖	39 ✖	40 $2\sqrt{10}$
41 ✖	42 ✖	43 ✖	44 $2\sqrt{11}$	45 $3\sqrt{5}$	46 ✖	47 ✖	48 $4\sqrt{3}$	49 7	50 $5\sqrt{2}$
51 ✖	52 $2\sqrt{13}$	53 ✖	54 $3\sqrt{6}$	55 ✖	56 $2\sqrt{14}$	57 ✖	58 ✖	59 ✖	60 $2\sqrt{15}$
61 ✖	62 ✖	63 $3\sqrt{7}$	64 8	65 ✖	66 ✖	67 ✖	68 $2\sqrt{17}$	69 ✖	70 ✖
71 ✖	72 $6\sqrt{2}$	73 ✖	74 ✖	75 $5\sqrt{3}$	76 $2\sqrt{19}$	77 ✖	78 ✖	79 ✖	80 $4\sqrt{5}$
81 9	82 ✖	83 ✖	84 $2\sqrt{21}$	85 ✖	86 ✖	87 ✖	88 $2\sqrt{22}$	89 ✖	90 $3\sqrt{10}$
91 ✖	92 $2\sqrt{23}$	93 ✖	94 ✖	95 ✖	96 $4\sqrt{6}$	97 ✖	98 $7\sqrt{2}$	99 $3\sqrt{11}$	100 10

Numbers and their square roots that can be SIMPLIFIED

Numbers from 1 to 100 that have a SQUARE NUMBER as a factor. Starting with the highest square numbers going to the lowest. (49, 36, 25, 16, 9, 4)

49	98 = 49 x 2 $\sqrt{98} = 7\sqrt{2}$	36	72 = 36 x 2 $\sqrt{72} = 6\sqrt{2}$	25	50 = 25 x 2 $\sqrt{50} = 5\sqrt{2}$	75 = 25 x 3 $\sqrt{75} = 5\sqrt{3}$	100 = 25 x 4 *
16	32 = 16 x 2	48 = 16 x 3	64 = 16 x 4 *	80 = 16 x 5	96 = 16 x 6		
9	18 = 9 x 2	27 = 9 x 3	36 = 9 x 4 *	45 = 9 x 5	54 = 9 x 6	63 = 9 x 7	72 = 9 x 8 *
	81 = 9 x 9 *	90 = 9 x 10	99 = 9 x 11				
4	8 = 4 x 2	12 = 4 x 3	16 = 4 x 4 *	20 = 4 x 5	24 = 4 x 6	28 = 4 x 7	32 = 4 x 8 *
	36 = 4 x 9 *	40 = 4 x 10	44 = 4 x 11	48 = 4 x 12 *	52 = 4 x 13	56 = 4 x 14	60 = 4 x 15
	64 = 4 x 16 *	68 = 4 x 17	72 = 4 x 18 *	76 = 4 x 19	80 = 4 x 20 *	84 = 4 x 21	88 = 4 x 22
	92 = 4 x 23	96 = 4 x 24 *	100 = 4 x 25 *		* means already done		

Surds - the first few square numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 .....

	Find the largest square number factor		
$\sqrt{32}$	$\sqrt{16 \times 2}$	$\sqrt{16} \times \sqrt{2}$	$4\sqrt{2}$
$\sqrt{8}$	$\sqrt{4 \times 2}$	$\sqrt{4} \times \sqrt{2}$	$2\sqrt{2}$
$\sqrt{50}$	$\sqrt{25 \times 2}$	$\sqrt{25} \times \sqrt{2}$	$5\sqrt{2}$
$\sqrt{27}$	$\sqrt{9 \times 3}$	$\sqrt{9} \times \sqrt{3}$	$3\sqrt{3}$
$\sqrt{48}$	$\sqrt{16 \times 3}$	$\sqrt{16} \times \sqrt{3}$	$4\sqrt{3}$
$\sqrt{98}$	$\sqrt{49 \times 2}$	$\sqrt{49} \times \sqrt{2}$	$7\sqrt{2}$
$\sqrt{12}$	$\sqrt{4 \times 3}$	$\sqrt{4} \times \sqrt{3}$	$2\sqrt{3}$
$\sqrt{20}$	$\sqrt{4 \times 5}$	$\sqrt{4} \times \sqrt{5}$	$2\sqrt{5}$
$\sqrt{18}$	$\sqrt{9 \times 2}$	$\sqrt{9} \times \sqrt{2}$	$3\sqrt{2}$
$\sqrt{24}$	$\sqrt{4 \times 6}$	$\sqrt{4} \times \sqrt{6}$	$2\sqrt{6}$
$\sqrt{75}$	$\sqrt{25 \times 3}$	$\sqrt{25} \times \sqrt{3}$	$5\sqrt{3}$

Simplify

1) $\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$

2) $4 \times \sqrt{8} = 4 \times 2\sqrt{2} = 8\sqrt{2}$

3) $\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6$

4) $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$

5) $3\sqrt{3} \times \sqrt{12} = 3\sqrt{3} \times 2\sqrt{3} = 6 \times \sqrt{9} = 6 \times 3 = 18$

6) $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{12}{3}} = \sqrt{4} = 2$

7) $\dfrac{\sqrt{54}}{\sqrt{2}}$ $\sqrt{\dfrac{54}{2}} = \sqrt{27} = 3\sqrt{3}$

8) $(\sqrt{3})^4$ $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}$
$= \sqrt{9} \times \sqrt{9}$
$= 3 \times 3$
$= 9$

9) $3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$

10) $\sqrt{2}(\sqrt{2} - 3) = \sqrt{2} \times \sqrt{2} - \sqrt{2} \times 3$
$= \sqrt{4} - 3\sqrt{2}$
$= 2 - 3\sqrt{2}$

Rules for Surds (Writing numbers using square roots)

Addition and Subtraction

1) $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$

2) $\sqrt{5} + \sqrt{5} + \sqrt{5} = 3\sqrt{5}$

3) $\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

4) $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

5) $\sqrt{2} + \sqrt{3} =$ cannot simplify

6) $4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3}$

7) $\sqrt{5} - \sqrt{2} =$ cannot simplify

8) $4\sqrt{3} + 2\sqrt{3} - \sqrt{3} + \sqrt{5} = 5\sqrt{3} + \sqrt{5}$

Multiplication

7) $\sqrt{3} \times \sqrt{5} = \sqrt{3\times5} = \sqrt{15}$

8) $\sqrt{2} \times \sqrt{7} = \sqrt{2\times7} = \sqrt{14}$.

9) $\sqrt{7} \times \sqrt{6} = \sqrt{7\times6} = \sqrt{42}$

10) $3 \times \sqrt{5} = 3\sqrt{5}$

11) $4 \times \sqrt{3} = 4\sqrt{3}$

12) $\sqrt{3} \times \sqrt{12} = \sqrt{3\times12} = \sqrt{36} = 6$

Division

13) $\frac{\sqrt{8}}{\sqrt{4}} = \sqrt{\frac{8}{4}} = \sqrt{2}$

14) $\frac{\sqrt{18}}{\sqrt{6}} = \sqrt{\frac{18}{6}} = \sqrt{3}$

15) $\frac{\sqrt{15}}{3} =$ cannot simplify

16) $\frac{\sqrt{18}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$

17) $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

Surds, the Key Skills

1 Finding the highest SQUARE NUMBER FACTOR.

The first few square numbers are 4, 9, 16, 25, 36, 49, 64, 81, 100

$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

2 ADDING AND TAKING LIKE TERMS

a) $3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$ b) $5\sqrt{3} - \sqrt{3} = 4\sqrt{3}$

c) $6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$ d) $5\sqrt{7} + 2\sqrt{7} = 7\sqrt{7}$

e) $6\sqrt{5} - 4\sqrt{5} = 2\sqrt{5}$ f) $3\sqrt{2} + 7\sqrt{2} - 6\sqrt{2} = 4\sqrt{2}$

Making them the same

g) $\sqrt{8} + \sqrt{18} =$

$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

h) $\sqrt{12} + \sqrt{48} =$

$2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$

i) $3\sqrt{8} + \sqrt{50} =$

$3 \times 2\sqrt{2} + 5\sqrt{2}$

$6\sqrt{2} + 5\sqrt{2} = 11\sqrt{2}$

j) $\sqrt{90} - \sqrt{40} =$

$3\sqrt{10} - 2\sqrt{10} = \sqrt{10}$

k) $3\sqrt{96} - 2\sqrt{24} =$

$3 \times 4\sqrt{6} - 2 \times 2\sqrt{6}$

$12\sqrt{6} - 4\sqrt{6} = 8\sqrt{6}$

l) $5\sqrt{63} + 2\sqrt{28} =$

$5 \times 3\sqrt{7} + 2 \times 2\sqrt{7}$

$15\sqrt{7} + 4\sqrt{7} = 19\sqrt{7}$

3 TIMES and DIVIDES

$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$ $2 \times \sqrt{3} \times \sqrt{6} = 2 \times \sqrt{3 \times 6} = 2 \times \sqrt{18}$

$$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

a) $\sqrt{5} \times \sqrt{10} = \sqrt{50} = 5\sqrt{2}$ b) $\sqrt{2} \times \sqrt{32} = \sqrt{64} = 8$

c) $\sqrt{6} \times \sqrt{10} = \sqrt{60} = 2\sqrt{15}$ d) $\sqrt{3} \times \sqrt{3} = \sqrt{9} = 3$

e) $\frac{\sqrt{20}}{\sqrt{10}} = \sqrt{2}$ f) $\frac{\sqrt{80}}{\sqrt{20}} = \sqrt{4} = 2$ g) $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{9} = 3$ h) $\frac{\sqrt{50}}{\sqrt{2}} = \sqrt{25} = 5$

4 MULTIPLYING BRACKETS

Single bracket $\sqrt{2}(1+\sqrt{2}) = \sqrt{2}\times 1 + \sqrt{2}\times\sqrt{2}$

a) $\sqrt{3}(2+\sqrt{3}) =$ $\sqrt{3}\times 2 + \sqrt{3}\times\sqrt{3} = 2\sqrt{3}+3$

b) $\sqrt{6}(\sqrt{6}-4) =$ $\sqrt{6}\times\sqrt{6} - 4\sqrt{6} = 6-4\sqrt{6}$

c) $\sqrt{2}(5+2\sqrt{2}) =$ $\sqrt{2}\times 5 + \sqrt{2}\times 2\times\sqrt{2} = 5\sqrt{2}+4$

d) $\sqrt{5}(2\sqrt{5}+\sqrt{3}) =$ $\sqrt{5}\times 2\times\sqrt{5} + \sqrt{5}\times\sqrt{3}$
$10+\sqrt{15}$

Two brackets $(\sqrt{2}+1)(3-\sqrt{2}) = \sqrt{2}\times 3 - \sqrt{2}\times\sqrt{2} + 1\times 3 - 1\times\sqrt{2}$

a) $(\sqrt{3}+1)(3+\sqrt{3}) =$ $\sqrt{3}\times 3 + \sqrt{3}\times\sqrt{3} + 1\times 3 + 1\times\sqrt{3}$
$= 3\sqrt{3} + 3 + 3 + \sqrt{3}$
$= 6+4\sqrt{3}$

b) $(2\sqrt{5}+1)(4+\sqrt{5}) =$ $2\sqrt{5}\times 4 + 2\sqrt{5}\times\sqrt{5} + 1\times 4 + 1\times\sqrt{5}$
$= 8\sqrt{5} + 10 + 4 + \sqrt{5}$
$= 14+9\sqrt{5}$

c) $(\sqrt{2}-3)(\sqrt{2}-1) =$ $\sqrt{2}\times\sqrt{2} - \sqrt{2}\times 1 - 3\times\sqrt{2} + 3\times 1$
$= 2 - \sqrt{2} - 3\sqrt{2} + 3$
$= 5-4\sqrt{2}$

REMEMBER

TRUE $\sqrt{2}\times\sqrt{4} = \sqrt{2\times 4} = \sqrt{8}$ $\qquad \dfrac{\sqrt{4}}{\sqrt{2}} = \sqrt{\dfrac{4}{2}} = \sqrt{2}$

FALSE $\sqrt{2}+\sqrt{4}$ *is not* $\sqrt{2+4}$ $\qquad \sqrt{2}-\sqrt{4}$ *is not* $\sqrt{2-4}$

Rationalising the denominator

When we rationalise the denominator of a fraction we remove the square root from the denominator, they can still be present in the numerator.

There are two situations

1) The denominator contains a single term. For example $\frac{2}{\sqrt{3}}$ or $\frac{5}{2\sqrt{3}}$ or $\frac{2+\sqrt{5}}{\sqrt{3}}$

In each of these cases multiply the original fraction by a new fraction which has the denominator of the original fraction as its numerator and denominator.

$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ then multiply them out and simplify $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$$\frac{5}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{10\sqrt{3}}{4\times 3} = \frac{5\sqrt{3}}{6}$$

$$\frac{2+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}(2+\sqrt{5})}{3} = \frac{2\sqrt{3}+\sqrt{15}}{3}$$

2) The denominator contains two terms. For example $\frac{2}{1+\sqrt{3}}$ or $\frac{5}{2\sqrt{3}-1}$

In each of these cases multiply the original fraction by a new fraction which has the denominator of the original fraction WITH ITS SIGN CHANGED as its numerator and denominator. This idea is based on the difference of two squares. (a – b)(a + b) = $a^2 - b^2$ $(1-\sqrt{3})(1+\sqrt{3}) = 1^2 - (\sqrt{3})^2 = 1 - 3 = -2$

$\frac{2}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$ then multiply them out and simplify $\frac{2\times 1 - 2\sqrt{3}}{1\times 1 - 1\times\sqrt{3} + \sqrt{3}\times 1 - \sqrt{3}\times\sqrt{3}}$

This simplifies to make $\frac{2-2\sqrt{3}}{1-3} = \frac{2-2\sqrt{3}}{-2} = \sqrt{3} - 1$

$$\frac{5}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1} = \frac{10\sqrt{3}+5}{2\sqrt{3}\times 2\sqrt{3} + 2\sqrt{3}\times 1 - 1\times 2\sqrt{3} - 1\times 1} = \frac{10\sqrt{3}+5}{11}$$

$\frac{5}{\sqrt{3}-2} \times \frac{\sqrt{3}*2}{\sqrt{3}+2} = -10-5\sqrt{3}$

Rationalise the denominator of these fractions

1) $\frac{3}{\sqrt{5}}$ 2) $\frac{7}{\sqrt{2}}$ 3) $\frac{5}{4\sqrt{6}}$ 4) $\frac{5}{4+\sqrt{6}}$ 5) $\frac{5}{\sqrt{3}-2}$ 6) $\frac{\sqrt{2}-1}{\sqrt{3}+3}$

$= \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ $= \frac{7}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{5}{4\sqrt{6}} \times \frac{4\sqrt{6}}{4\times 6}$ $\frac{5}{4+\sqrt{6}} \times \frac{4-\sqrt{6}}{4-\sqrt{6}}$ $\frac{\sqrt{2}-1}{\sqrt{3}+3} \times \frac{\sqrt{3}-3}{\sqrt{3}-3}$

$= \frac{3\sqrt{5}}{5}$ $= \frac{7\sqrt{2}}{2}$ $= \frac{20\sqrt{6}}{96}$ $= \frac{4-\sqrt{6}}{2}$ $= \frac{\sqrt{3}+3\sqrt{2}-\sqrt{6}-3}{6}$

$= \frac{5}{24}\sqrt{6}$

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