SIMULTANEOUS EQUATIONS

Solve these Simultaneous Equations

1 $3x + 2y = 8$

Add $\dfrac{5x - 2y = 8}{8x = 16}$

$x = 2$

$3x + 2y = 8$
$3 \times 2 + 2y = 8$
$6 + 2y = 8 \quad 2y = 2 \quad y = 1$

Check
$5x - 2y = 8$
$5 \times 2 - 2 \times 1$
$ 8 = 8 \checkmark$

2 $2c + 3d = 13$

TAKE $\dfrac{2c - d = 1}{4d = 12}$

$d = 3$

$2c + 3d = 13$
$2c + 3 \times 3 = 13$
$2c + 9 = 13$
$2c = 4$
$c = 2$

Check.
$2c - d = 1$
$2 \times 2 - 3$
$1 = 1 \checkmark$

3 $3a - b = 5$

TAKE $\dfrac{5a - b = 9}{-2a = -4}$

$a = \dfrac{-4}{-2} = 2$

$3a - b = 5$
$3 \times 2 - b = 5$
$6 - b = 5$
$b = 1$

check.
$5a - b = 9$
$5 \times 2 - 1$
$9 = 9 \checkmark$

4 $-3c + 3d = -9$

Add $\dfrac{3c - d = 11}{2d = 2}$

$d = 1$

$3c - d = 11$
$3c - 1 = 11$
$3c = 12$
$c = 4$

check.
$-3c + 3d = -9$
$-3 \times 4 + 3 \times 1$
$-12 + 3$
$-9 = -9 \checkmark$

1) The sign of a term is the sign in front of it.

2) To cancel a term with

OPPOSITE SIGNS **ADD**

SAME SIGNS **TAKE**

3) The rules for negative numbers

$3 - 5 = -2$

$3 + -1 = 3 - 1 = 2$

$3 - -1 = 3 + 1 = 4$

$-9 - 11 = -20$

$-2 - -4 = -2 + 4 = 2$

4) Find the values of both letters.

5) Check by substituting that the answers are correct

①

Solve these simulataneous equations

1)

$2x + y = 7$

$3x - y = 3$

Add _____

$5x = 10$

$x = 2$

$x = 2, y = 3$

$2x + y = 7$

$2 \times 2 + y = 7$

$4 + y = 7$

$y = 3$

2)

$5x + 3y = 23$

$5x - y = 19$

TAkE _____

$4y = 4$

$y = 1$

$x = 4, y = 1$

$5x + 3y = 23$

$y = 1$ $5x + 3 \times 1 = 23$

$5x + 3 = 20$

$5x = 20$

$x = 4$

3) $2x + 3y = 8$ times by 2

$4x + 2y = 8$

$2x + 3y = 8$

$y = 2$ $2x + 3 \times 2 = 8$

$2x + 6 = 8$

$2x = 2$

$x = 1$

$4x + 6y = 16$

$4x + 2y = 8$

TAkE _____

$4y = 8$

$y = 2$

$x = 1, y = 2$

4) $2x + 3y = 12$

$5x - y = 13$ times by 3

$2x + 3y = 12$

$x = 3$ $2 \times 3 + 3y = 12$

$6 + 3y = 12$

$3y = 6$

$y = 2$

$2x + 3y = 12$

$15x - 3y = 39$

Add _____

$17x = 51$

$x = 3$

$x = 3, y = 2$

5) $7x + 3y = 26$ times by 2

$4x + 2y = 16$ times by 3

$4x + 2y = 16$

$x = 2$

$4 \times 2 + 2y = 16$

$8 + 2y = 16$

$2y = 8$

$y = 4$

$14x + 6y = 52$

$12x + 6y = 48$

TAkE _____

$2x = 4$

$x = 2$

$x = 2, y = 4$

6) $5x + 3y = 14$ times by 4

$6x - 4y = -6$ times by 3

$5x + 3y = 14$

$x = 1$

$5 \times 1 + 3y = 14$

$5 + 3y = 14$

$3y = 9$

$y = 3$

$20x + 12y = 56$

$18x - 12y = -18$

Add _____

$38x = 38$

$x = 1$

$x = 1, y = 3$

②

1) $5x + 3y = 19$
   $2x + 3y = 13$
   $x = 2 \quad y = 3$

2) $5x - 2y = 1$
   $x - 2y = -3$
   $x = 1 \quad y = 2$

3) $3x - 2y = 7$
   $2x + 2y = 8$
   $x = 3 \quad y = 1$

4) $2x + 4y = 8$
   $-2x + 5y = 1$
   $x = 2 \quad y = 1$

5) $2x + y = 7$
   $x + 2y = 5$
   $x = 3 \quad y = 1$

6) $2x - 2y = 6$
   $3x + y = 13$
   $x = 4 \quad y = 1$

7) $5x - y = 7$
   $4x + 2y = 14$
   $x = 2 \quad y = 3$

8) $3x - 4y = 7$
   $2x - 2y = 6$
   $x = 5 \quad y = 2$

9) $4x + 2y = 8$
   $3x + 5y = 13$
   $x = 1 \quad y = 2$

10) $2x + 3y = 7$
    $3x - 2y = 4$
    $x = 2 \quad y = 1$

11) $3x - 3y = 3$
    $4x - 2y = 6$
    $x = 2 \quad y = 1$

12) $7x - 2y = 1$
    $4x + 5y = 19$
    $x = 1 \quad y = 3$

③

Simultaneous Equations - write simualtaneous equations for each question and solve them

1 t c
2 cups of tea and 3 cakes cost £4.20

$2t + 3c = 420$

4 cups of tea and 1 cake costs £3.90

$4t + c = 390$

How much is a cup of tea?   75p
How much is a cake?   90p

2 x y
4 cans and 3 crisps costs £3.40

$4x + 3y = 340$

3 cans and 6 crisps costs £4.05

$3x + 6y = 405$

How much is a can?   55p
How much is a bag of crisps?   40p

3   3 nuts and 4 bolts cost 69p $3n + 4b = 69$

1 nut and 2 bolts cost 31p $n + 2b = 31$

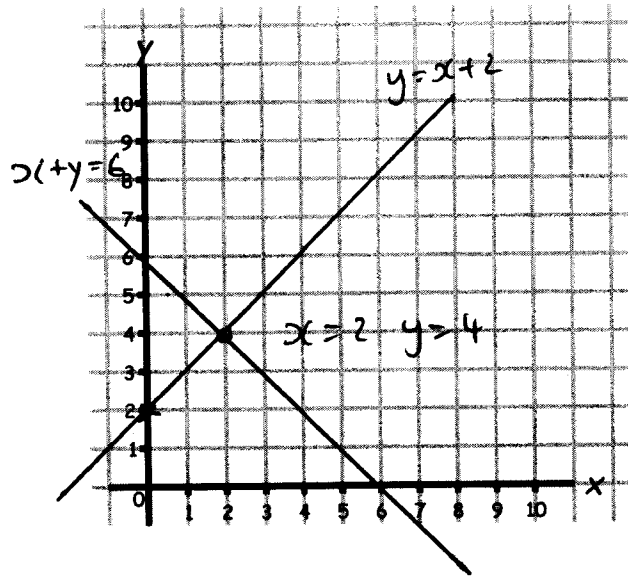1 nut costs?   7p
1 bolt costs?   12p

4   4 screws and 2 nails weigh 17 grams. $4s + 2n = 17$

3 screws and 5 nails weigh 32 grams. $3s + 5n = 32$

1 screw weighs?   1.5g
1 nail weighs?   5.5g
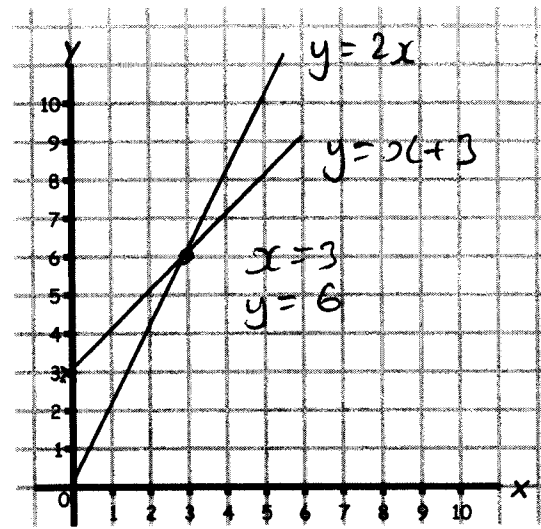
④

Solving Simultaneous Equations Graphically

$y = x + 2$

x	0	1	2	3	4	5	6	7	8
y	2	3	4	5	6	7	8	9	10

$x + y = 6$

x	0	1	2	3	4	5	6
Y	6	5	4	3	2	1	0

The answer

x = __2__ , y = __4__

$y = 2x$

x	0	1	2	3	4	5
Y	0	2	4	6	8	10

$y = x + 3$

x	0	1	2	3	4	5	6
Y	3	4	5	6	7	8	9

The answer

x = __3__ , y = __6__
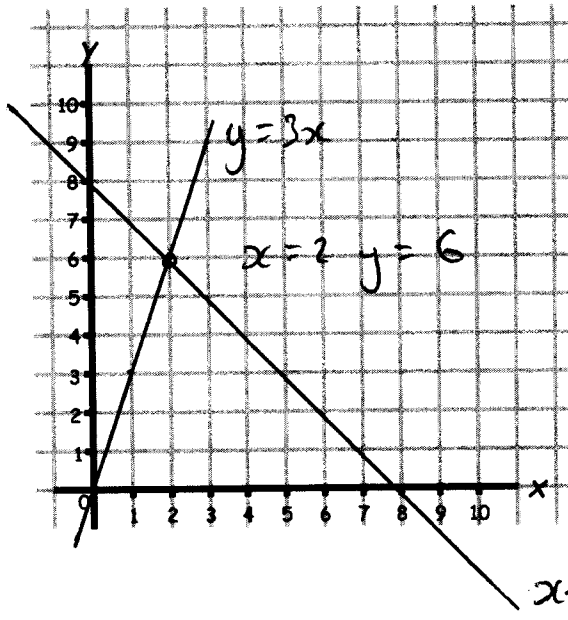
$x + y = 8$

x	0	1	2	3	4	5	6	7	8
Y	8	7	6	5	4	3	2	1	0

$y = 3x$

x	0	1	2	3
Y	0	3	6	9

The answer

x = __2__ , y = __6__
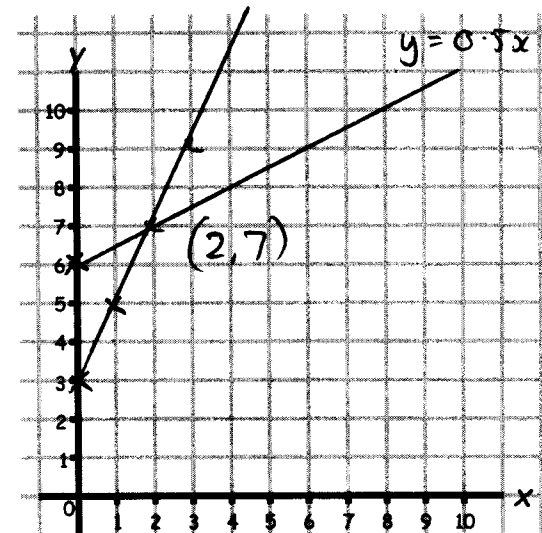
$y = 0.5x + 6$

x	0	1	2	3	4	5	6	7	8
Y	6	6.5	7	7.5	8	8.5	9	9.5	10

$y = 2x + 3$

x	0	1	2	3
Y	3	5	7	9

The answer

x = __2__ , y = __7__

⑤

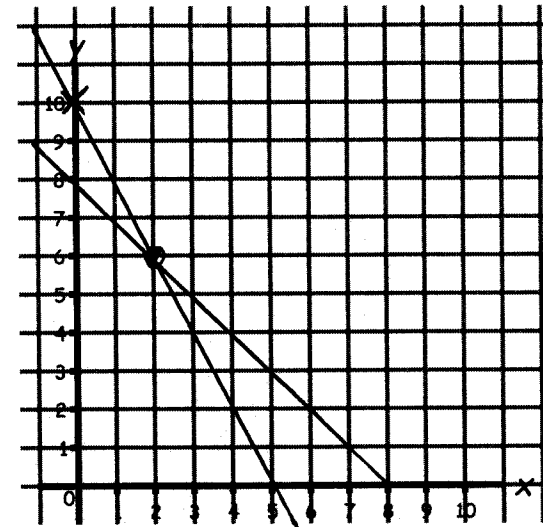# Solving Simultaneous Equations Graphically

$y + 2x = 8$

when $x = 0$, $y = 8$ plot(0,8)

when $y = 0$, $x = 4$ plot (4, 0)

$3y + 2x = 12$

when $x = 0$, $y = 4$ plot(0,4)
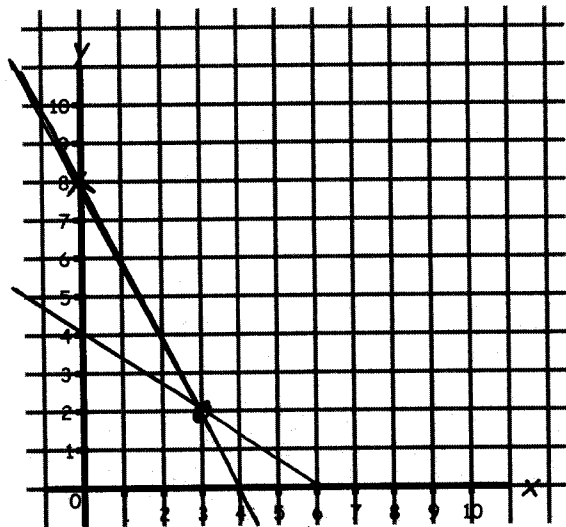
when $y = 0$, $x = 6$ plot (6, 0)

$y + 2x = 8$
$3y + 2x = 12$ Answer $x = 3$ $y = 2$
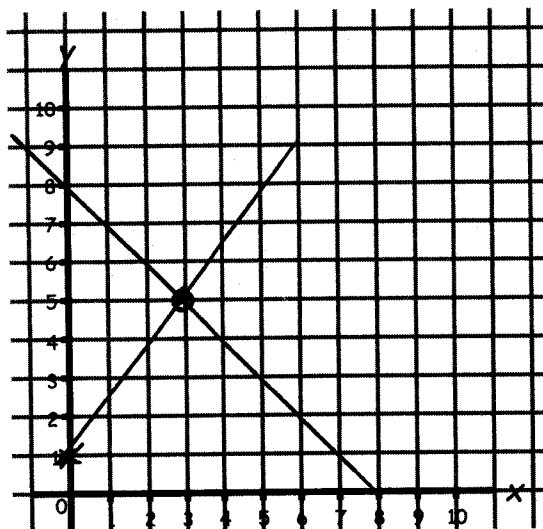

$y + 2x = 10$

when $x = 0$, $y = 10$ plot(0,10)

when $y = 0$, $x = 5$ plot (5, 0)

$y + x = 8$

when $x = 0$, $y = 8$ plot(0,8)

when $y = 0$, $x = 8$ plot (8, 0)

$y + 2x = 10$
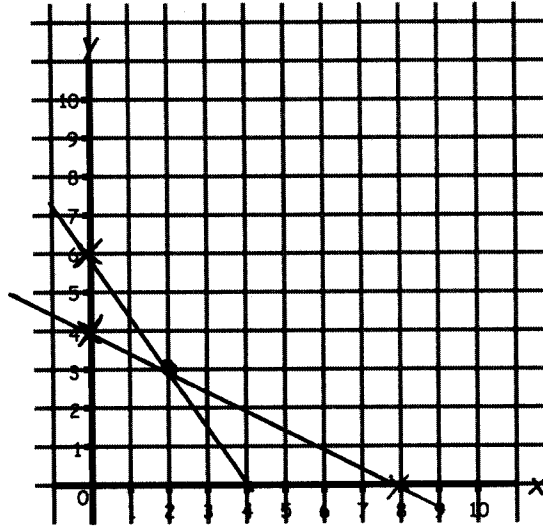$y + x = 8$ Answer $x = 2$ $y = 6$

$y + x = 8$

when $x = 0$, $y = 8$ plot(0,8)

when $y = 0$, $x = 8$ plot (8, 0)

$3y = 4x + 3$

when $x = 0$, $y = 1$ plot(0,1)

when $x = 6$, $y = 9$ plot (6,9)

$y + x = 8$
$3y = 4x + 3$ Answer $x = 3$ $y = 5$

$2y + x = 8$

when $x = 0$, $y = 4$ plot(0,4)
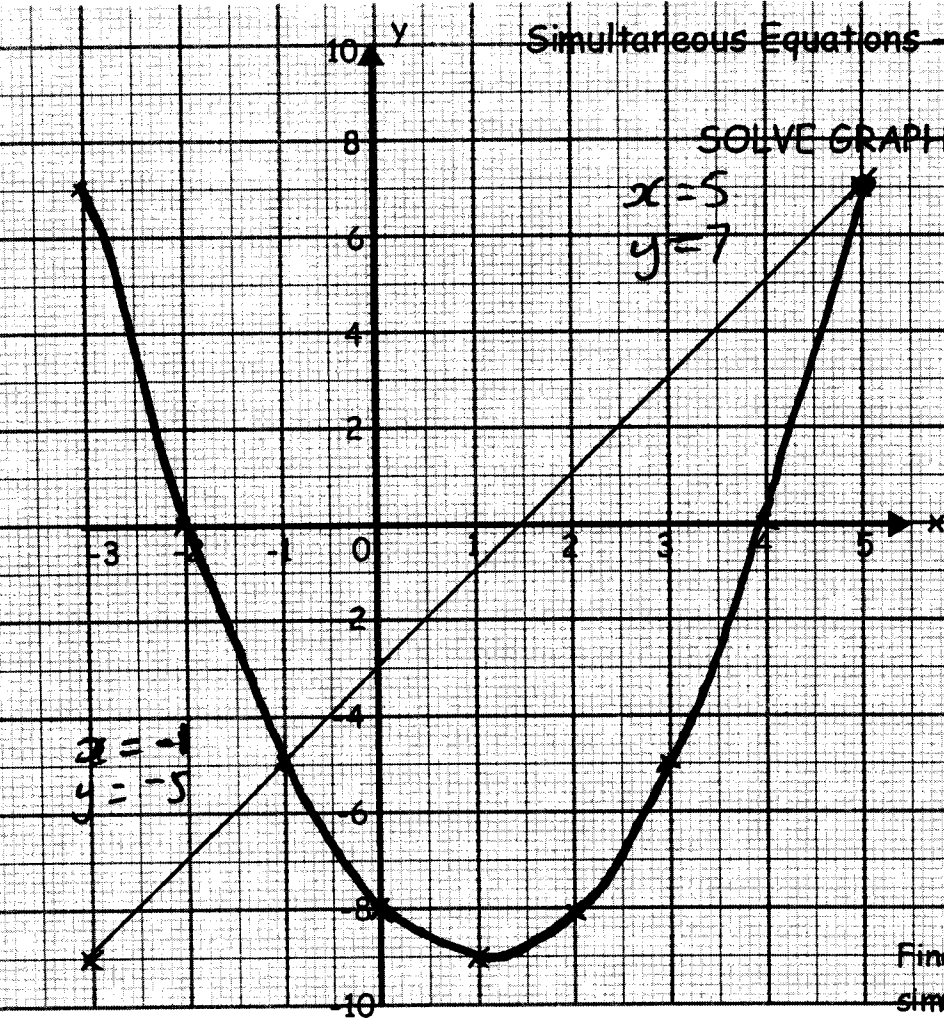
when $y = 0$, $x = 8$ plot (8, 0)

$2y + 3x = 12$

when $x = 0$, $y = 6$ plot(06)

when $y = 0$, $x = 4$ plot (4, 0)

$2y + x = 8$
$2y + 3x = 12$ Answer $x = 2$ $y = 3$

⑥

Simultaneous Equations - one quadratic one linear

SOLVE GRAPHICALLY $y = x^2 - 2x - 8$ and $y = 2x - 3$

$x = 5$
$y = 7$

Draw the graph of $y = x^2 - 2x - 8$

x	-3	-2	-1	0	1	2	3	4	5
y	7	0	-5	-8	-9	-8	-5	0	7

Plot the points on the graph and join them with a smooth curve.

Draw the graph of $y = 2x - 3$

x	-3	-2	-1	0	1	2	3	4	5
y	-9	-7	-5	-3	-1	1	3	5	7

$x = -1$
$y = -5$

Find the coordinates of the points where the graphs cross. This is the solution to the simultaneous equations.

For a given x coordinate they both have the same y coordinate, therefore they cross

$x = -1 \quad y = -5$ and $x = 5 \quad y = 7$

Another way of writing this is $\quad x^2 - 2x - 8 = 2x - 3$

the curve   the straight line

What values of x make the curve and the line have the same y value?

⑦

Simultaneous Equations – one quadratic, one linear

SOLVE ALGEBRAICALLY

Example – Solve these pair of simultaneous equations $y = x^2 - 2x - 8$ and $y = 2x - 3$

As both equations are of the form "y =", put them equal to each other.

$x^2 - 2x - 8 = 2x - 3$ By doing this we are finding the value of x that makes them have the same y value

Rearrange this to make a quadratic that is equal to 0. $x^2 - 4x - 5 = 0$

Solve this by factorising $(x - 5)(x + 1) = 0$ x = 5 or x = -1

For each value of x find the value of y. You can use either the curve or the straight line for this.

When x = 5   y= 2 x 5 – 3 = 7

When x = -1   y= 2 x -1 – 3 = -5

The final answer is x = 5, y = 7 and x = -1, y = -5

Have a go at solving these ones

1) $y = x^2 + x + 1$ and $y = 6x + 15$

$x^2 + x + 1 = 6x + 15$

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$ $x = -2 \quad y = 3$

$x = 7 \quad y = 57$

2) $y = x^2 + 5x$ and $y = x + 5$

$x^2 + 5x = x + 5$

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$ $x = 1 \quad y = 6$

$x = -5 \quad y = 0$

3) $y = x^2 + 2x + 2$ and $y = 3x + 4$

$x^2 + 2x + 2 = 3x + 4$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$ $x = 2 \quad y = 10$

$x = -1 \quad y = 1$

4) $y = -x^2 + x + 6$ and $y = x + 5$

$-x^2 + x + 6 = x + 5$

$0 = x^2 - 1$

$(x - 1)(x + 1) = 0$ $x = 1 \quad y = 6$

$x = -1 \quad y = 4$

⑧