RATIO and PROPORTION

Ratio

Bag	Number of White and Black Balls	White : Black	Black : White	Fraction White	Fraction Black
1	○ ○ ○ ●	3 : 1	1 : 3	3/4	1/4
2	○ ●	1 : 1	1 : 1	1/2	1/2
3	● ○ ● ●	1 : 3	3 : 1	1/4	3/4
4	○ ○ ○ ○ ●	4 : 1	1 : 4	4/5	1/5
5	○ ● ○ ○ ●	3 : 2	2 : 3	3/5	2/5
6	● ● ○	1 : 2	2 : 1	1/3	2/3
7	○ ○ ○ ○ ○ ●	5 : 1	1 : 5	5/6	1/6
8	○ ● ○	2 : 1	1 : 2	2/3	1/3
9	○ ● ● ●●	1 : 4	4 : 1	1/5	4/5
10	○ ● ●○ ●	2 : 3	3 : 2	2/5	3/5

11 ○ ● ○ ○ ●

a) What is the ratio of white:black? 3 : 2
b) What is the ratio of black:white? 2 : 3
c) What fraction of the circles are white? 3/5
d) What fraction of the circles are black? 2/5
e) If this pattern were repeated so that there were 9 white circles, how many black circles would there be? 6
f) If this pattern were repeated so that there were 4 black circles, how many white circles would there be? 6

12 The ratio in a circle pattern of white:black is 3:1. ○ ○ ○ ●

a) What fraction of the circles are white? 3/4
b) What fraction of the circles are black? 1/4
c) If this pattern were repeated so that there were 9 white circles, how many black circles would there be? 3
d) If this pattern were repeated so that there were 4 black circles, how many white circles would there be? 12

13 The ratio in a circle pattern of white:black is 1:4. ○ ● ● ● ●

a) What fraction of the circles are white? 1/5
b) What fraction of the circles are black? 4/5
c) If this pattern were repeated so that there were 4 white circles, how many black circles would there be? 16
d) If this pattern were repeated so that there were 8 black circles, how many white circles would there be? 2

Ratio Yellow to Red 2:1

no.	Yellow	Red	Total
1	2	1	3
3	6	3	9
2	4	2	6
5	10	5	15
4	8	4	12
6	12	6	18
14	28	14	42

no. column is the number of times you need to repeat the basic starting pattern.

	1	2	3	4	5	6
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
red	R	R	R	R	R	R

Ratio Red to Yellow is 1 : 2
What fraction is red $\frac{1}{3}$ is yellow $\frac{2}{3}$

Ratio Yellow to Red 3:1

no.	Yellow	Red	Total
1	3	1	4
3	9	3	12
2	6	2	8
5	15	5	20
4	12	4	16
6	18	6	24
14	42	14	56

	1	2	3	4	5	6
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
red	R	R	R	R	R	R

Ratio Red to Yellow is 1 : 3
What fraction is red $\frac{1}{4}$ is yellow $\frac{3}{4}$

Ratio Yellow to Red 3:2

no.	Yellow	Red	Total
1	3	2	5
2	6	4	10
4	12	8	20
5	15	10	25
3	9	6	15
6	18	12	30
13	39	26	65

	1	2	3	4	5	6
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
red	R	R	R	R	R	R
red	R	R	R	R	R	R

Ratio Red to Yellow is 2 : 3
What fraction is red $\frac{2}{5}$ is yellow $\frac{3}{5}$

Ratio Yellow to Red 4:1

no.	Yellow	Red	Total
1	4	1	5
3	12	3	15
2	8	2	10
5	20	5	25
4	16	4	20
6	24	6	30
9	36	9	45

	1	2	3	4	5	6
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
yellow	Y	Y	Y	Y	Y	Y
red	R	R	R	R	R	R

Ratio Red to Yellow is 1 : 4
What fraction is red $\frac{1}{5}$ is yellow $\frac{4}{5}$

Splitting in a given ratio

1 Split £40 in the ratio 3:5

Ratio		Number of parts
3	5	8
3×5=15	5×5=25	40

↓ x 5

40÷8=5

2 Split £35 in the ratio 2:3

Ratio		Number of parts
2	3	5
14	21	35

↓ x 7

3 Split £36 in the ratio 2:7

Ratio		Number of parts
2	7	9
8	28	36

↓ x 4

4 Split £16 in the ratio 1:7

Ratio		Number of parts
1	7	8
2	14	16

↓ x 2

5 John and Sally split some money in the ratio 2:5. John receives £10. How much does Sally get? How much money is split altogether?

Ratio		Number of parts
John	Sally	
2	5	7
10	25	35

↓ x 5

6 John and Sally split some money in the ratio 3:2. Sally receives £10. How much does John get? How much money is split altogether?

Ratio		Number of parts
John	Sally	
3	2	5
15	10	25

↓ x 5

7 John and Sally split some money in the ratio 4:3. John receives £16. How much does Sally get? How much money is split altogether?

Ratio		Number of parts
John	Sally	
4	3	7
16	12	28

↓ x 4

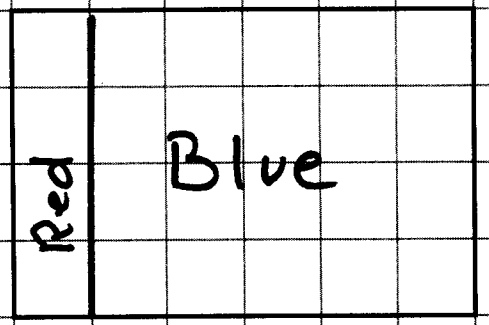
You have to colour the rectangles in the following ratios. You must choose which rectangle to use. You must define the ratio. eg. for the ratio 3:4

Red = 3

Blue = 4

Ratios to use 3:4, 1:4:5, 1:5, 1:4, 3:5 and 1:2

7 10 6 5 8 3



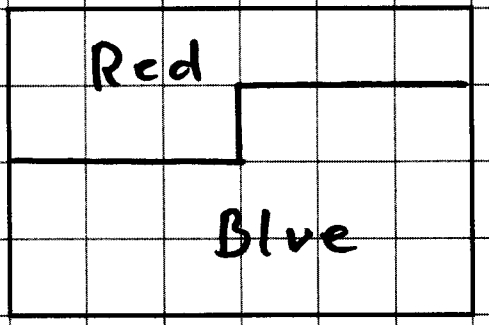
Number of squares 24

Ratio 1:5

Definition Red : Blue

1 : 5

×4 ↓ 4 : 20



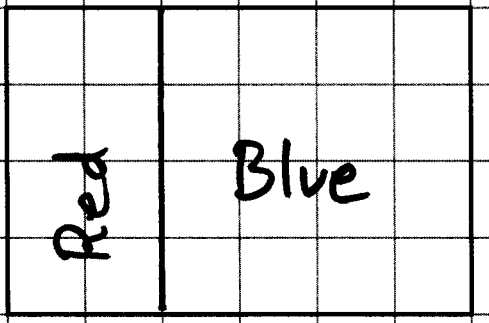
Number of squares 24

Ratio 3:5

Definition Red : blue

3 : 5

×3 ↓ 9 : 15



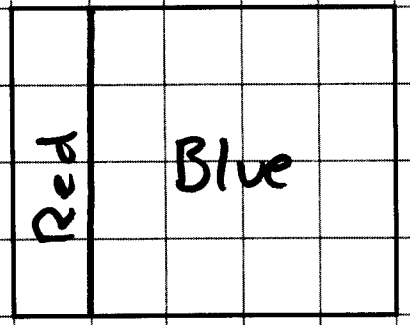
Number of squares 24

Ratio 1:2

Definition Red : blue

1 : 2

×8 ↓ 8 : 16



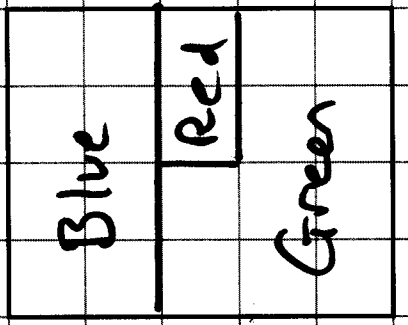
Number of squares 20

Ratio 1:4

Definition Red : blue

1 : 4

×4 ↓ 4 : 16



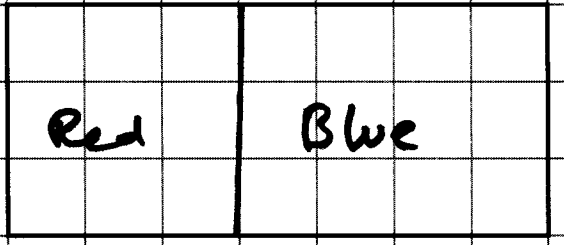
Number of squares 20

Ratio 1:4:5

Definition Red : blue : Green

1 : 4 : 5

×2 ↓ 2 : 8 : 10



Number of squares 21

Ratio 3:4

Definition Red : blue

3 : 4

×3 ↓ 9 : 12

1st Count how many squares in each drawing

2nd Add the numbers in each ratio.

3rd Check if ratio number is (4) a factor of number of squares

24 ÷ 6 = 4 1:5

24 ÷ 8 = 3 3:5

24 ÷ 3 = 8 1:2

21 ÷ 7 = 3 3:4.

20 ÷ 10 = 2 1:4:5

20 ÷ 5 = 4 1:4

Ratio

1) Simplify these ratios

a) 24 : 6 → 4 : 1 b) 35 : 10 → 7 : 2 c) 1200 : 500 → 12 : 5 d) 24 : 20 → 6 : 5 e) 9 : 6 → 3 : 2

2) Divide £48 in the ratio 5 : 3

5 : 3 → 8 ↓ ×6 → £30 : £18 → 48

3) Divide £120 in the ratio 8 : 7.

8 : 7 → 15 ↓ ×8 → £64 : £56 → 120

4) Peter divides some sweets between Kerry and Jenny in the ratio 4 : 3. Kerry receives 24 sweets.

a) How many sweets does Jenny receive? 18

b) How many sweets did Peter share out? 42

	K : J	Total
	4 : 3	7
×6 ↓	24 : 18	42

5) Sam has £15. The ratio of the money Sam has to the money Alice has is 5 : 2. How much money does Alice have? £6

	S : A	Total
	5 : 2	7
↓ ×3	15 : 6	21

6) Leroy is 14 and Samuel is 8.

a) Write in the simplest form the ratio of their ages in the order Leroy : Samuel.

14 : 8 = 7 : 4

Uncle Charles split divides £110 between Leroy and Samuel in the ratio of their ages.

	L : S	Total
	7 : 4	11
	70 : 40	110

b) How much does Leroy receive? £70

In 4 years time Uncle Charles divides some more money between the two boys in the ratio of their current ages. Samuel receives £48, how much does Leroy receive? £72

L : S	
18 : 12	
3 : 2	5
72 : 48	↓ ×24

7) Write the following ratios in the form 1 : n

a) 2 : 6 → 1 : 3 b) 4 : 10 → 1 : 2·5 c) 5 : 2 → 1 : 0·4 d) 2 : 11 → 1 : 5·5

8) Khalid wants to split some counters in the ratio 2 : 5. What is the smallest possible number of counters he needs? 7

9) The ratio of dogs to cats is 3 : 2. The ratio of cats to rabbits is 5 : 2.

What is the ratio of dogs to rabbits? 15 : 4

D : C	C : R
3 : 2	5 : 2
×5 ↓ 15 : 10	10 : 4 ↓ ×2

10) Len has 25 slices of bread and 11 slices of cheese. He is going to make sandwiches. Each sandwich requires 2 slices of bread and 1 slice of cheese. How many sandwiches can he make? 11

11) The ratio of a : b is 2 : 1. Which of these statements is true, a = 2b or b = 2a? a = 2b

12) Ki has some blue and red counters. The ratio blue to red is 11 : 6. Are more than 35% of his counters red?

Fraction red = $\frac{6}{17}$ = 0·353 35·3% Yes

Comparing Quantities SHOW YOUR WORKING OUT

1) Which is bigger $\frac{1}{2}$ *or* $\frac{1}{4}$? $\frac{1}{2} = \frac{2}{4}$ $\frac{2}{4}$ is bigger than $1/4$

$\frac{1}{2} = 0.5$ $\frac{1}{4} = 0.25$ or $1/2 = 50\%$ $\frac{1}{4} = 25\%$.

2) Which is bigger $\frac{2}{3}$, 0.64 *or* 68% ? 68% is the biggest.

$\frac{2}{3} = 0.\dot{6}$ $68\% = 0.68$

3) John and Sally have some red and blue counters.

For John, the ratio red : blue is 1:2 For Sally, the ratio red : blue is 2:3

Who has the highest proportion of blue counters?

John $2/3$ of his counters are blue, for Sally it is $3/5$

$2/3 = 0.\dot{6}$ $3/5 = 0.6$ John has higher proportion.

4) In tests Sandra scored 12 out of 30 and Brian score 34 out of 80. Who did better?

Sandra $\frac{12}{30} = 12 \div 30 = 0.4 = 40\%$ Brian $\frac{34}{80} = 34 \div 80 = 0.425$ 42.5%

Brian

5) I can buy 5 pens for 85p at Shop A or 7 pens for £1.26 at Shop B. Which shop is better value?

Shop A $85 \div 5 = 17p$ per pen B $126 \div 7 = 18p$ per pen

Shop A

6) I can buy 4 apples for £1.08 at Shop A or 3 apples for 78p at Shop B. Which shop is better value?

A $108 \div 4 = 27p$ B $78 \div 3 = 26p$ B

7) 30g of Ready Salted crisps contain 12 g of fat. 25g of Cheese and Onion crisps contain 9.5 g of fat.

Which flavor crisps contain the least amount of fat?

Ready Salted $\frac{12}{30} = 0.4 = 40\%$ Cheese & Onion $\frac{9.5}{25} = 0.38 = 38\%$.

Cheese and Onion.

8) A car does 240 miles on 20 litres of petrol. A van does 363 miles on 33 litres. Which vehicles goes the furthest on 1 litre of petrol?

Car $\frac{240}{20} = 12$ miles per litre Van $\frac{363}{33} = 11$ m/L

CAR.

9) 0.65 litres of cooking oil weighs 0.78 kg. How much will 1.5 litres weigh?

1 litre weighs $\frac{0.78}{0.65} = 1.2$ kg $1.5 \times 1.2 = 1.8$ kg

10) In a test Dylan scored 42%. Harry got two fifths of the total marks available. Who did best?

Harry $\frac{2}{5} = 0.4 = 40\%$ Dylan

Direct Proportion

1) 5 pens cost £1.45. How much do 7 pens cost?

1 pen $145 \div 5 = 29p$

7 pens $7 \times 29p = £2.03$

2) 6 rulers cost £2.10. How much do 11 rulers cost?

1 ruler $210 \div 6 = 35p$

11 $11 \times 35 = £3.85$

3) 3 apples cost £1.29. How much do 4 apples cost?

1 $129 \div 3 = 43p$

4 $43 \times 4 = £1.72$

4) 7 oranges cost £2.59. How much do 8 oranges cost?

1 $259 \div 7 = 37p$

8 $37 \times 8 = £2.96$

5) 6 bananas cost £2.46. How much will 7 bananas cost?

1 $246 \div 6 = 41$

7 $41 \times 7 = £2.87$

6) A car travels 72 miles on 8 litres of petrol. How far will it go on 10 litres of petrol?

1 $72 \div 8 = 9$

10 $10 \times 9 = 90$ miles

7) A car drives at a fixed speed. It goes 80 miles in 2 hours. How far would it go in 5 hours if it travelled at the same speed?

1 hour $80 \div 2 = 40$

5 hours $5 \times 40 = 200$ miles

8) A tap fills a 20 litre tank in 5 minutes. How many minutes would it take to fill a 36 litre tank?

In 1 minute $20 \div 5 = 4$ litres of water. $36 \div 4 = 9$ minutes

or $5 \div 20 = 0.25$ minutes for 1 litre $0.25 \times 36 = 9$ minutes.

9) £5 is worth $6.50. How many dollars is £8 worth?

£1 $6.50 \div 5 = \$1.30$ £8 $8 \times 1.30 = \$10.40$

10) 0.3 litres of coke costs 24p. How much will 1.4 litres of coke cost?

1 litre costs $\frac{24}{0.3} = 80p$.

1.4 litres $80 \times 1.4 = £1.12$

Direct Proportion

1) 5 pens cost £20. C = Cost in £ n = number of pens

a) 1 pen costs? $20 \div 5 = 4$

b) Write the formula C = 4 x n

c) What is the cost of pens 7? $C = 4 \times 7 = £28$

d) How many pens can I buy for £48? $48 = 4 \times n \quad n = \frac{48}{4} = 12$

2) £2 is worth $4 D = Dollars P = pounds

a) £1 is worth how many dollars? 2

b) Write the formula D = 2 x P

c) How many dollars for £5? $D = 2 \times 5 = \$10$

d) How many pounds for $14? $14 = 2 \times P \quad P = \frac{14}{2} = £7$

3) 10 books cost £20 C = cost in £ B = number of books

a) 1 book costs? 2

b) Write the formula C = 2 x B

c) Find C, when B = 7 $C = 2 \times 7 = £14$

d) Find B, when C = £132 $132 = 2 \times B \quad B = \frac{132}{2} = 66$

4) 3 calculators cost £12. C = Cost in £ n = number of calculators

a) What is the cost of 1 calculator? 4

b) Write the formula C = 4 x n

c) What is the cost of 4 calculators? $C = 4 \times 4 = £16$

d) How many calculators can I buy with £32? $32 = 4 \times n \quad n = \frac{32}{4} = 8$

5) 5 coffees cost £12.50. C = Cost in £ n = number of coffees

a) What is the cost of 1 coffee? 2·50

b) Write the formula C = 2·50 x n

c) What is the cost of 4 coffee? $C = 2{\cdot}50 \times 4 = £10$

d) How many coffees can I buy with £35? $35 = 2{\cdot}50 \times n \quad n = \frac{35}{2{\cdot}50} = 14$

Inverse Proportion.

1) Some sweets are shared between a group of people. If there are 9 people they each receive 4 sweets. Total number of sweets $9 \times 4 = 36$

a) How many sweets do they each get if there are 8 people? $36 \div 8 = 4.5$

b) How many people are there if each person gets 2 sweets? $36 \div 2 = 18$

c) S = sweets and P = people. Write a formula S = $\frac{36}{P}$

2) Some counters are shared between a group of people. If there are 6 people they each receive 10 counters. Number of counters $6 \times 10 = 60$

a) How many counters do they each get if there are 5 people? $60 \div 5 = 12$

b) How many people are there if each person gets 4 counters? $60 \div 4 = 15$

c) C = counters and P = people. Write a formula C = $\frac{60}{P}$

3) Some money is shared between a group of people. If there are 5 people they each receive £20. TOTAL Money = $5 \times 20 = £100$

a) How much do they each get if there are 8 people? $100 \div 8 = £12.50$

b) How many people are there if each person gets £6.25? $100 \div 6.25 = 16$

c) M = money and P = people. Write a formula M = $\frac{100}{P}$

4) Some stickers are shared between a group of people. If there are 12 people they each receive 4 stickers. $12 \times 4 = 48$ Stickers

a) How many stickers do they each get if there are 8 people? $48 \div 8 = 6$

b) How many people are there if each person gets 2 stickers? $48 \div 2 = 24$

c) S = stickers and P = people. Write a formula S = $\frac{48}{P}$

5) Sally is going to paint her house. If she does it on her own it will take her 30 hours. Assuming everyone works at the same rate

a) How long will it take 6 people? $30 \div 6 = 5$

b) The job took 2 and a half hours. How many people were there? $30 \div 2.5 = 12$

c) T = time taken and P = people. Write a formula T = $\frac{30}{P}$.

Inverse and Direct Proportion

1) It takes 3 workers 8 hours to build a wall. Inverse $3 \times 8 = 24$ hours of work.

a) How long would it take 4 workers to build the same wall? $24 \div 4 = 6$

b) If the wall was built in 2 hours, how many workers were there? $24 \div 2 = 12$

2) It takes Kelly 25 seconds to run 200m. Assuming she runs at the same pace, Direct

a) How long will it take her to run 56m? $56 \div 8 = 7$ secs 25 seconds = 200m

b) How far would she run in a minute? 1 second = $\frac{200}{25} = 8$ m

$60 \times 8 = 480$m

3) £4 is worth $6. direct £4 = $6

a) How many $s is £7? £1 = $\frac{6}{4} = \$1.50$ $7 \times 1.50 = \$10.50$

b) How many £s is $25? $25 \div 1.50 = £16.67$

4) John has some money to share equally between prize winners. If there were 5 prize winners they would each receive £20. Inverse $5 \times 20 = £100$

a) If there were 8 prize winners, how much would they each receive? $100 \div 8 = £12.50$

b) Each prize winner received £2.50. How many winners were there?

$100 \div 2.50 = 40$

5) Shantae shares here sweets equally between her friends. When there are 6 of them they receive 8 sweets each. Inverse $6 \times 8 = 48$ sweets

a) How many sweets each if there are 4 of them? $48 \div 4 = 12$

b) How many people are there if they get 2 sweets each? $48 \div 2 = 24$.

6) 0.24 kg of metal has a volume of 0.15 m^3. Direct

$0.24\text{kg} = 0.15\ m^3$

a) What is the volume of 1.3 kg of metal? 1.3×0.625 $0.8125\ m^3$ 1 kg = $\frac{0.15}{0.24} = 0.625\ m^3$

b) What is the mass of 0.5 m^3 of metal? $0.5 \div 0.625$

0.8 kg

(10)

Proportion

Direct Proportion

Formula	Proportional relationship
$y = 5x$	y is proportional to x
$y = 4x^2$	y is proportional to x^2
$y = 0.25x^3$	y is proportional to x^3
$y = 0.2\sqrt{x}$	y is proportional to $\sqrt{x}$

1) y is proportional to x. When x = 4, y = 20. $20 \div 4 = 5$
Find the formula that connects x and y. $y = 5x$
Find y when x = 7. $y = 5 \times 7 = 35$
Find x when y = 65. $65 = 5x \quad x = \frac{65}{5} = 13$

2) y is proportional to x squared. When x = 3, y = 27. $3^2 x ? = 27 \quad 9x ? = 27 \quad ? = \frac{27}{9} = 3$
Find the formula that connects x and y. $y = 3x^2$
Find y when x = 6. $y = 3 \times 6^2 = 3 \times 36 = 108$
Find x when y = 243. $243 = 3x^2 \quad x^2 = 81 \quad x = \sqrt{81} = 9$

3) y is proportional to x cubed. When x = 4, y = 12.8.
Find the formula that connects x and y. $y = 0.2 \times x^3$ $\frac{12.8}{4^3} = 0.2$
Find y when x = 6. $y = 0.2 \times 6^3 = 43.2$
Find x when y = 200. $200 = 0.2 \times x^3 \quad x^3 = 200 \div 0.2 = 1000 \quad x = \sqrt[3]{1000} = 10$

4) y is proportional to the square root of x. When x = 4, y = 12.
Find the formula that connects x and y. $y = 6\sqrt{x}$ $\frac{12}{\sqrt{4}} = \frac{12}{2} = 6$
Find y when x = 9. $y = 6 \times \sqrt{9} = 6 \times 3 = 18$
Find x when y = 24.

$24 = 6 \times \sqrt{x}$

$\sqrt{x} = \frac{24}{6} = 4$

$x = 4^2 = 16$

Inverse Proportion

Formula	Proportional relationship
$y = 20/x$	y is inversely proportional to x
$y = 5/x^2$	y is inversely proportional to x^2
$y = 10/x^3$	y is inversely proportional to x^3
$y = 0.6/\sqrt{x}$	y is inversely proportional to $\sqrt{x}$

5) y is inversely proportional to x. When x = 4, y = 6. $4 \times 6 = 24$.
Find the formula that connects x and y. $y = \frac{24}{x}$
Find y when x = 9. $y = \frac{24}{9} = 2.6$
Find x when y = 24. $24 = \frac{24}{x} \quad x = \frac{24}{24} = 1$

6) y is inversely proportional to x^2. When x = 2, y = 3. $3 \times 2^2 = 12$
Find the formula that connects x and y. $y = \frac{12}{x^2}$
Find y when x = 4. $y = \frac{12}{4^2} = 0.75$
Find x when y = 0.12. $0.12 = \frac{12}{x^2} \quad 0.12x^2 = 12 \quad x^2 = \frac{12}{0.12} = 100 \quad x = 10$

7) y is inversely proportional to x^3. When x = 2, y = 6.
Find the formula that connects x and y. $y = \frac{48}{x^3}$ $6 \times 2^3 = 48$
Find y when x = 4. $y = \frac{48}{4^3} = 0.75$
Find x when y = 0.384. $0.384 = \frac{48}{x^3} \quad x^3 = \frac{48}{0.384} = 125 \quad x = 5$

8) y is inversely proportional to the square root of x. When x = 4, y = 6.
Find the formula that connects x and y. $y = \frac{12}{\sqrt{x}}$ $6 \times \sqrt{4} = 12$
Find y when x = 9. $y = \frac{12}{\sqrt{9}} = 4$
Find x when y = 3.

$3 = \frac{12}{\sqrt{x}} \quad \sqrt{x} = \frac{12}{3} = 4$

$x = 16$

Proportion

Qu.	Formula	Direct or Inverse?	Find y given x	Find x given y
1	$y = \frac{20}{x}$	Inverse	x = 2 y = 10	y = 5 x = 4
2	y = 8x	Direct	x = 3 y = 24	y = 36 x = 4·5
3	y = 0.2x	D	x = 10 y = 2	y = 4 x = 20
4	$y = \frac{10}{x}$	I	x = 5 y = 2	y = 1 x = 10
5	$y = \frac{4}{x}$	I	x = 0.5 y = 8	y = 2 x = 2
6	y = 2.4x	D	x = 5 y = 12	y = 2.4 x = 1
7	y = 10x	D	x = 12 y = 120	y = 72 x = 7·2
8	$y = \frac{0.2}{x}$	I	x = 2 y = 0·1	y = 10 x = 50
9	y = 160x	D	x = 0.5 y = 80	y = 40 x = 0·25
10	$y = \frac{1}{x}$	I	x = 2 y = 0·5	y = 5 x = 0·2