PROBABILITY

Page	Description
1	Balls in a bag. Writing probabilities as fractions. Using the language of probability
2	Number spinners. Writing probabilities as fractions. Using the language of probability
3	Writing probabilities as fractions. Arrange events in order of likelihood
4	Probability Scales
5	Combined Events. Sample space diagrams
6	Writing probabilities as fractions for single and combined events. Arrange events in order of likelihood Sheet 1
7	Writing probabilities as fractions for single and combined events. Arrange events in order of likelihood Sheet 2
8	Relative Frequency
9	Relative Frequency from experiments
10	Relative Frequency from experiments
11	Mutually Exclusive
12	Conditional Probability
13	Probablilty using Venn Diagrams and two way tables
14	Use Venn diagrams to find probabilities
15	Use Venn diagrams to find probabilities
16	Tree diagrams with and without replacement. 2 types of balls in a bag
17	Tree diagrams with and without replacement. 2 types of balls in a bag
18	Tree diagrams with and without replacement. 3 types of buttons in a bag
19	Tree diagrams not using selecting from a group of objects

Probability

Bag	Number of White and Black Balls	Probability of choosing Black	Probability of choosing White
1	○ ○ ○ ●	$\frac{1}{4}$	$\frac{3}{4}$
2	○ ○ ● ●	$\frac{2}{4} = \frac{1}{2}$	$\frac{2}{4} = 1/2$
3	● ○ ● ●	3/4	1/4
4	○ ○ ○ ○ ●	1/5	4/5
5	○ ● ○ ○ ●	2/5	3/5
6	● ● ● ●	$\frac{4}{4} = 1$	$\frac{0}{4} = 0$
7	○ ○ ○ ○ ○ ●	1/6	5/6
8	○ ●	1/2	1/2
9	○ ● ● ●●	4/5	1/5
10	○	$0/1 = 0$	$1/1 = 1$

1 Which bag are you most likely to choose a black ball from? 6

2 Which bag are you most likely to choose a white ball from? 10

3 Which bags give you a fifty chance of picking white or black? 2, 8

4 Which bag is it impossible to choose a white from? 6

5 Which bag is it impossible to choose a black from? 10

6 Are you likely or unlikely to choose a black ball from bag 1? unlikely

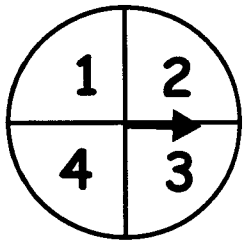
7 Are you likely or unlikely to choose a black ball from bag 3? likely

8 Are you likely or unlikely to choose a black ball from bag 7? unlikely

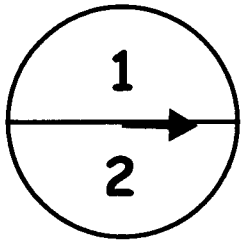
9 Are you likely or unlikely to choose a black ball from bag 9? likely

10 Are you likely or unlikely to choose a black ball from bag 5? unlikely

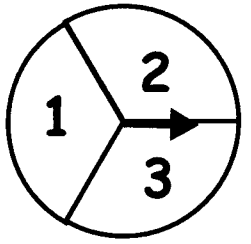
There are 8 fair spinners. The arrow is spun.



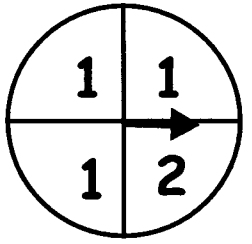
Spinner A



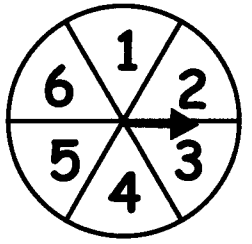
Spinner B



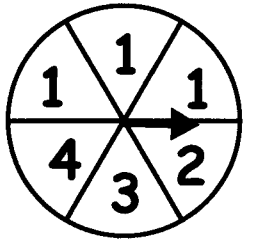
Spinner C



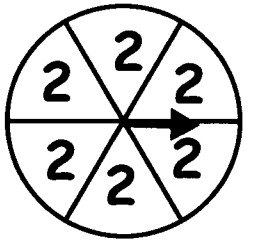
Spinner D



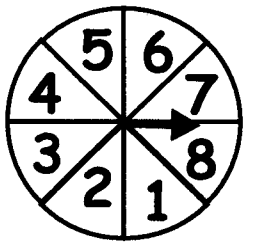
Spinner E



Spinner F



Spinner G



Spinner H

1 Which spinner are you most likely to get a 2 on? G

2 Which spinner are you least likely to get a 1 on? G

3 What is the probability of getting a 1 on each spinner?

A 1/4 C 1/3 E 1/6 G 0

B 1/2 D 3/4 F $\frac{3}{6} = \frac{1}{2}$ H 1/8

4 Which spinner are you most likely to get a 1 on? D

5 Which spinner could you use a dice? E

6 Which spinner could you use instead of a coin? B

7 Which spinner are you most likely to get an 8 on? H

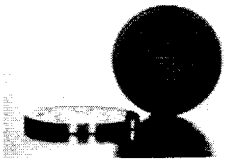
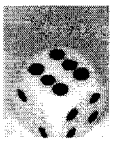
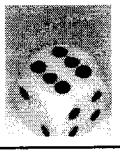
8 Which spinner has the probability of getting a 2 as $\frac{1}{3}$? C

9 Which two spinners have the probability of getting a 2 equal to $\frac{1}{4}$? A and D

10 Which two spinners have the probability of getting a 1 equal to $\frac{1}{2}$? B and F

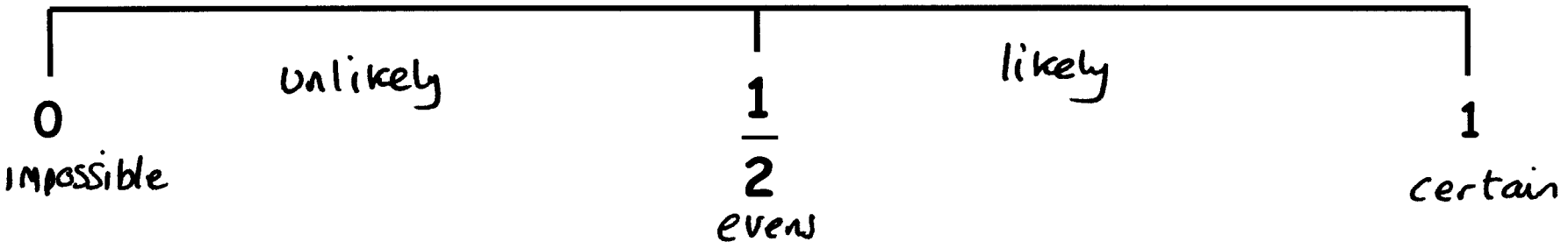
There are 12 cards each describes an event. In each card write the probability of that event happening. Write the 12 letters A to L in order of likelihood of the event happening. Least to most.

Least Likely C, B, H, F, J, L, I, G, D, E Most Likely
K A

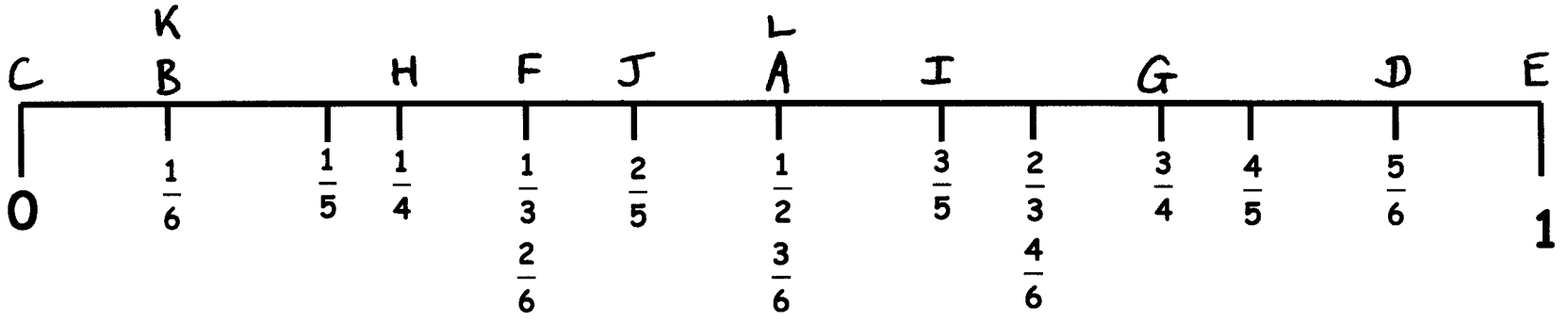
A $\frac{1}{2}$ Throwing a tail with a coin 	B $\frac{1}{6}$ Throwing a six with a dice 	C 0 Throwing a seven with a dice 
D 5/6 Throwing a number more than 1 on a dice 	E 1 Throwing a number less than 7 on a dice 	F $\frac{2}{6}=\frac{1}{3}$ Throwing a number in the 3 times table on a dice 
G 3/4 Choosing a white ball from the bag 	H 1/4 Choosing a black ball from the bag 	I 3/5 Choosing a white ball from the bag 
J 2/5 Choosing a black ball from the bag 	K $\frac{1}{6}$ Throwing a number in the 4 times table on a dice 	L $\frac{3}{6}=\frac{1}{2}$ Throwing an even number with a dice 

Probability Scales

Place the words certain, even, impossible, likely and unlikely in the correct place on this probablility scale



Using the events from worksheet 3 . Place the letters A to L in the correct place on the scale



COMBINED EVENTS using a sample space diagram

1) 2 dice are thrown. The scores showing are added togther to make a total.

2	5

total = 7

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Of the totals you can make, the most likely total is?

7

Of the totals you can make, the least likely total is?

2 and 12

P(1) = 0 P(4) = 3/36 P(7) = 6/36 P(10) = 3/36

P(2)= 1/36 P(5)= 4/36 P(8)= 5/36 P(11)= 2/36

P(3)= 2/36 P(6)= 5/36 P(9)= 4/36 P(12)= 1/36

2) 2 dice are thrown. The smaller number is taken from the larger.

2	5

total = 5 - 2 = 3

-	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The most likely total is?

0 1

The least likely total is?

5

P(0) = 6/36 P(2) = 8/36 P(4) = 4/36

P(1)= 10/36 P(3)= 6/36 P(5)= 2/36

24 cards each describes an event. In each card write the probability of that event happening. Write the 24 letters A to X in order of likelihood, least to most. Write one below the other if there are any that have the same probability

Least Likely — Most Likely

M N O P Q R $\frac{1}{36}$ | C D E F G H K L U W $\frac{1}{6}=\frac{6}{36}$ | J T $1/3=\frac{12}{36}$ | X $\frac{7}{18}=\frac{14}{36}$ | V $\frac{4}{9}=\frac{16}{36}$ | A B I S $1/2=\frac{18}{36}$

A 1 dice Throw an **odd** number $\frac{3}{6}=\frac{1}{2}$ $\frac{18}{36}$	B 1 dice Throw an **even** number $\frac{3}{6}=\frac{1}{2}$ $\frac{18}{36}$	C 1 dice Throw a 1 with a dice $\frac{1}{6}$ $\frac{6}{36}$
D 1 dice Throw a 2 with a dice $\frac{1}{6}$ $\frac{6}{36}$	E 1 dice Throw a 3 with a dice $\frac{1}{6}$ $\frac{6}{36}$	F 1 dice Throw a 4 with a dice $\frac{1}{6}$ $\frac{6}{36}$
G 1 dice Throw a 5 with a dice $\frac{1}{6}$ $6/36$	H 1 dice Throw a 6 with a dice $\frac{1}{6}$ $6/36$	I 1 dice Throw a **multiple of 2** with a dice $\frac{3}{6}=\frac{1}{2}$ $18/36$
J 1 dice Throw a **multiple of 3** with a dice $2/6=1/3$ $\frac{12}{36}$	K 1 dice Throw a **multiple of 4** with a dice $\frac{1}{6}$ $\frac{6}{36}$	L 2 dice Throw a **any double** with two dice $\frac{6}{36}=\frac{1}{6}$ $6/36$

M 2 dice Throw a **double** **1** with two dice $\frac{1}{36}$	N 2 dice Throw a **double** **2** with two dice $\frac{1}{36}$	O 2 dice Throw a **double** **3** with two dice $\frac{1}{36}$
P 2 dice Throw a **double** **4** with two dice $\frac{1}{36}$	Q 2 dice Throw a **double** **5** with two dice $\frac{1}{36}$	R 2 dice Throw a **double** **6** with two dice $\frac{1}{36}$
S 2 dice When added the total is a **multiple of 2** $\frac{18}{36} = \frac{1}{2}$ $\frac{18}{36}$	T 2 dice When added the total is a **multiple of 3** $\frac{12}{36} = \frac{1}{3}$ 12/36	U 2 dice When added the total is **7** $\frac{6}{36} = \frac{1}{6}$ 6/36
V 2 dice When added the total is 6, 7 or 8 $\frac{16}{36} = \frac{4}{9}$ $\frac{16}{36}$	W 2 dice When added the total is 2, 3, 11 or 12 $\frac{6}{36} = \frac{1}{6}$ 6/36	X 2 dice When added the total is 4, 5, 9 or 10 $\frac{14}{36} = \frac{7}{18}$ $\frac{14}{36}$

Probablility

1) The probability of getting a colour on a spinner is given in this table.

Red	Blue	Green	Yellow
0.1	0.3	0.4	0.2

What is the probability of choosing

a) Red or blue? $0.1 + 0.3 = 0.4$

b) Not Green? $1 - 0.4 = 0.6$

c) Yellow or blue? $0.2 + 0.3 = 0.5$

The spinner is spun 80 times. How many of each colour would you expect?

d) Red $0.1 \times 80 = 8$ e) Blue $0.3 \times 80 = 24$ f) Green $0.4 \times 80 = 32$ g) Yellow $0.2 \times 80 = 16$

2) Peter and Sally recorded the colour of cars at a road junction.

Red	Red	Silver	Green	Black	Blue	Orange	Green	Red	Red
Silver	Grey	White	White	Red	Blue	Red	Black	Grey	Blue

a) What is the relative frequency for the colour red? $6/20 = 3/10 = 0.3$

b) Greg observed 240 cars at the same junction. How many red cars would you expect him to see? $0.3 \times 240 = 72$

3)

Drink	Total
Coke	52
Lemonade	87
Tango	30
Sprite	31
	200

A group of students were asked about their favourite drink.

a) What is the relative frequency for Lemonade? $\frac{87}{200} = 87 \div 200 = 0.435$

b) If 700 students were asked, how many would you expect to choose Lemonade? $700 \times 0.435 = 304.5$ has to be a whole number because number of students = 305

4) Barny throws a dice 200 times and gets 50 sixes.

a) What is the relative frequency of throwing a 6? $50 \div 200 = 0.25$

b) What is the theoretical probability of throwing a 6 with a dice? $1/6 = 1 \div 6 = 0.16$

c) Do you think Barny's dice is biased? Yes 0.25 is quite a bit bigger than 0.16 and he threw it quite a large (200) number of times.

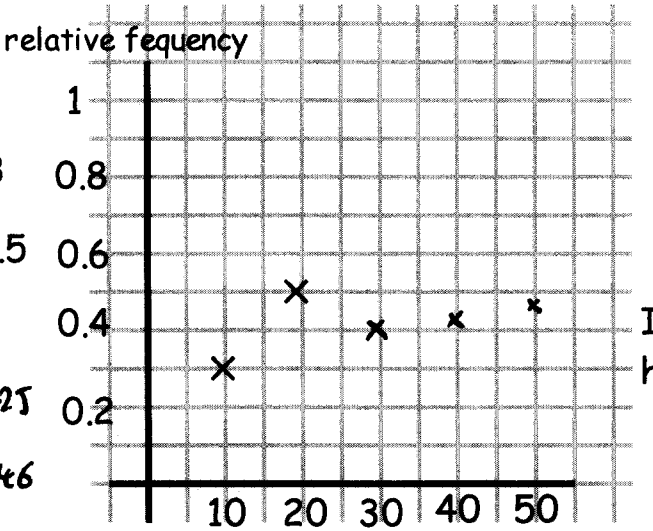
5) A bag contains blue, green or red counters only. P(blue) = 0.3 and P(green) = 0.5. What is P(red)?

$1 - 0.5 - 0.3 = 0.2$

Complete the tables and graphs. Answer the questions

Throwing a head with a coin

10 Throws	Heads	Total heads	Total Throws	Relative Frequency
THTTHTTHTT	3	3	10	3÷10 = 0.3
HTHHTHHHTH	7	10	20	10÷20 = 0.5
TTTTHTHTTT	2	12	30	12÷30 = 0·4
HHHTHHTTTT	5	17	40	17÷40 = 0·425
HHTHHHHTTT	6	23	50	23÷50 = 0·46



Expected (Theoretical)

$P(\text{Head}) = \frac{1}{2} = 0.5$

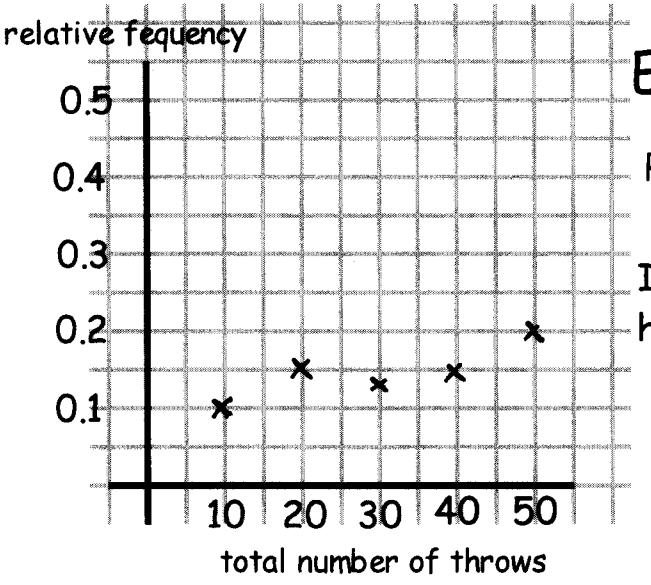
If a coin were thrown 1000 times how many heads would you expect?

500

If a coin were thrown 1000 times, using your best experimental result, how many heads would you expect? $0.46 \times 1000 = 460$

Throwing a six with a dice

10 Throws	6's	Total 6's	Total Throws	Relative Frequency
4452152263	1	1	10	0·1
4341216632	2	3	20	0·15
1242555623	1	4	30	$0.1\dot{3}$
4554246655	2	6	40	0·15
2632363626	4	10	50	0·2



Expected (Theoretical)

$P(6) = \frac{1}{6} = 1 \div 6 = 0.1\dot{6}$

If a dice were thrown 1200 times how many heads would you expect?

200

If a coin were thrown 1200 times, using your best experimental result, how many heads would you expect? $0.2 \times 1200 = 240$

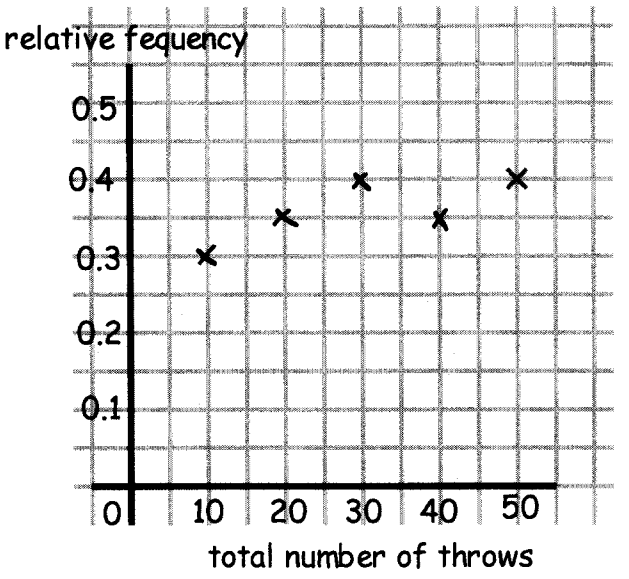
NOTE: the best estimate from the table comes from the one with the most trials.

Relative Frequency

A dice is rolled in blocks of 10 throws. The number of sixes thrown are recorded.

First block of 10. 6,1,2,3,6,3,6,5,4,4 Second block of 10. 6,1,2,3,6,3,6,5,6,4

6's in that block of 10	Total number of 6's	Total Throws	Relative Frequency
3	3	10	$3 \div 10 = 0.3$
4	7	20	$7 \div 20 = 0.35$
5	12	30	$12 \div 30 = 0.4$
14-12 2	$0.35 \times 40 = 14$	40	0.35
20-14 6	$0.4 \times 50 = 20$	50	0.4 ← read from the graph.



Which is the best relative frequency of throwing a 6 from this experimentnd why? the last one (most trials) 0.4.

Using the results from this experiment, how many 6's would you expect to throw in 2000 throws of the dice?

$2000 \times 0.4 = 800$

Is the dice biased? Yes

Theoretical probability for a 6 on a dice $= \frac{1}{6} = 0.1\dot{6}$

0.4 is much larger than $0.1\dot{6}$

Mutually Exclusive - means that there is no overlap, that means there is nothing that can be in both events.

If two events A and B are mutually exclusive then the probability of A or B is the probability of A plus the probability of B.

1) The probability of choosing a Ford from a car park is 0.3. The probability of choosing a Fiat from the same car park is 0.4.

a) Are these events mutually exclusive? Yes

b) What is the probability of choosing a Ford OR a Fiat from the car park? 0·3+0·4 =0·7

c) Why isn't the answer to part b) 1? There must be some other makes.

d) Are there more Fords or Fiats in the car park? Fiats

2) The probability of choosing a Ford from a car park is 0.3. The probability of choosing a Red car from the same car park is 0.5.

a) Are these events mutually exclusive? No

b) What is the probability of choosing a Ford OR a Red car from the car park? cannot say

c) Are there more Fords or Red cars in the car park? Red cars

3) The probability of choosing a Ford from a car park is 0.3.

a) What is the probability of choosing a vehicle that is not a Ford? 0·7

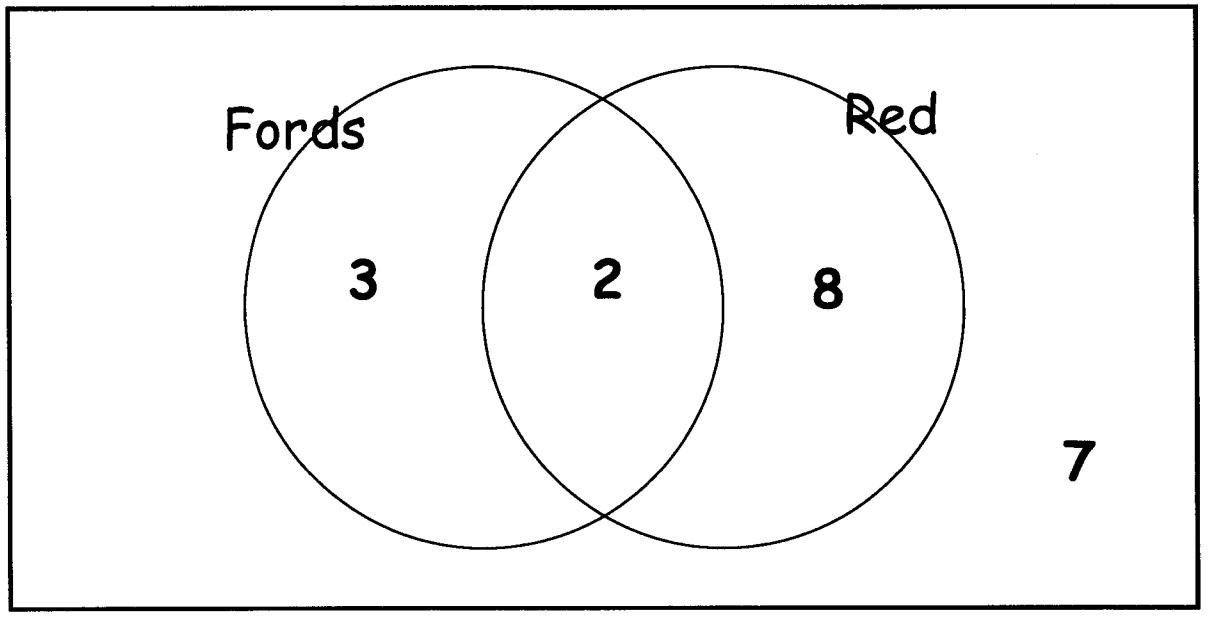
b) What is the probability of choosing a Ford OR a non Ford the car park? 1

c) Why is the answer to part b) 1? Ford and non Ford covers all the possible options.

d) Are there more Fords or non Fords in the car park?

Non Fords

Conditional Probability



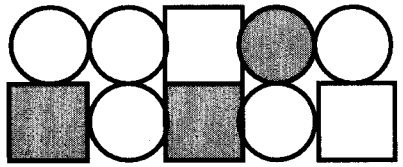
Cars in a car park

What is the probability of choosing. These five questions are not conditional probability

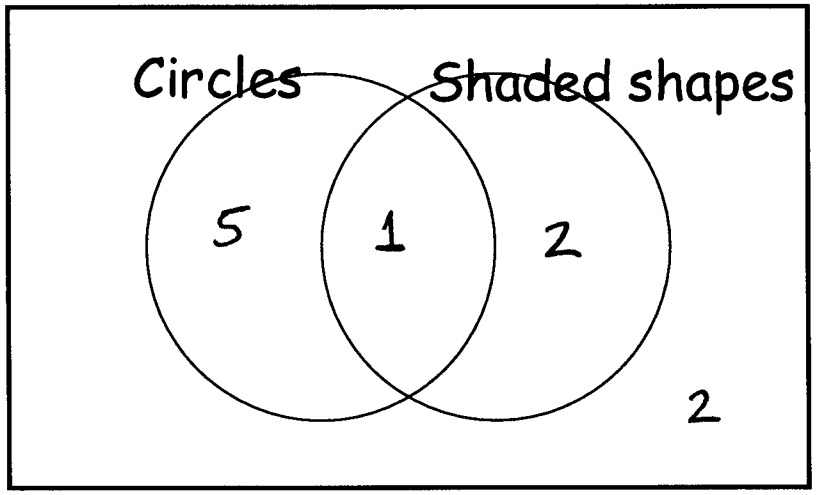
1) A red car $10/20 = 1/2$
2) A Ford $5/20 = 1/4$
3) A red Ford $2/20 = 1/10$
4) a non Ford $15/20 = 3/4$
5) A red car or a Ford $13/20$

These two questions are conditional probability, because the cars are being chosen from the cars that meet certain conditions and not chosen from all the cars.

6) Given that the car is red, what is the probability it is a Ford? $2/10 = 1/5$
7) Given that the car is a Ford, what is the probability it is red? $2/5$



Using these 10 shapes fill in the Venn Diagram. Write the number of shapes that will be in each section.



Fill in the Two Way table

	Circles	Squares
Shaded	1	2
Unshaded	5	2

What is the probability of choosing

1) A square $4/10 = 2/5$
2) A shaded circle $1/10$
3) A circle $\frac{6}{10} = 3/5$

4) Given that the shape is a circle, what is the probability it is shaded? $1/6$

5) Given that the shape is shaded, what is the probability it is a circle? $1/3$

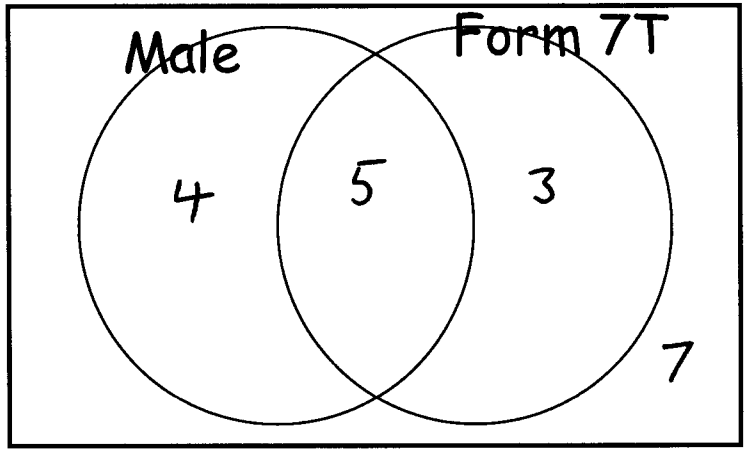
In a Year 7 Science group students are either in forms 7T or 7S.

What is the probability of choosing a student that is

6) Male $9/19$
7) In Form 7S $11/19$
8) A female in Form 7T $3/19$
9) Given that the student is female, what is the probability that they are in 7T? $3/10$
10) Given that the student is in 7T, what is the probability that they are female? $3/8$

	Male	Female
Form 7T	5	3
Form 7S	4	7

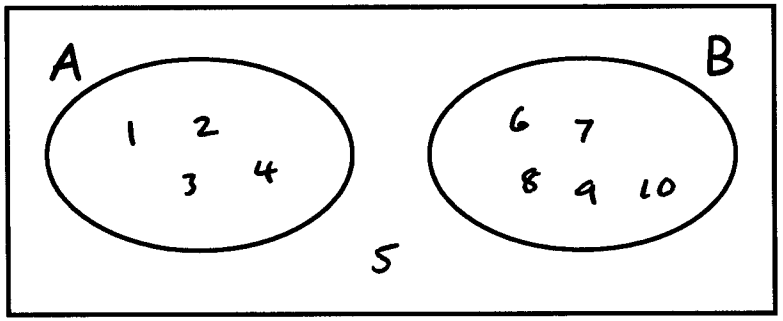
Fill in the Venn Diagram



VENN DIAGRAMS Consider the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 only (THE UNIVERSAL SET symbol ξ)

Put the numbers 1 to 10 in each Venn Diagram and answer the questions

ξ

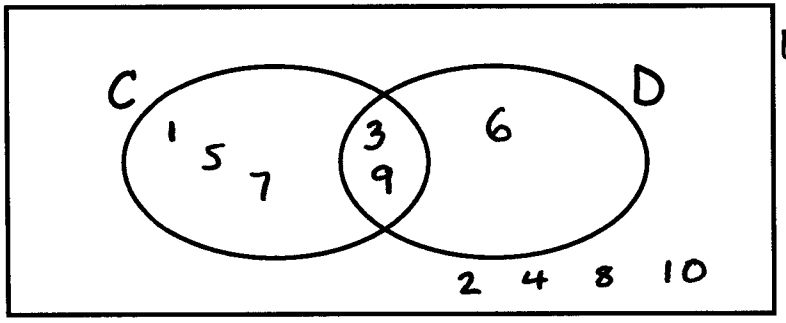


Event A = numbers less than 5

Event B = numbers more than 5

$P(A) = 4/10 = 2/5$
$P(B) = 5/10 = 1/2$
$p(A \text{ or } B) = p(A \cup B) = 9/10$
$P(A \text{ and } B) = P(A \cap B) = 0$
$P(A \text{ given } B) = 0$
$P(B \text{ given } A) = 0$
$P(\text{not } A) = P(A') = 6/10$

ξ

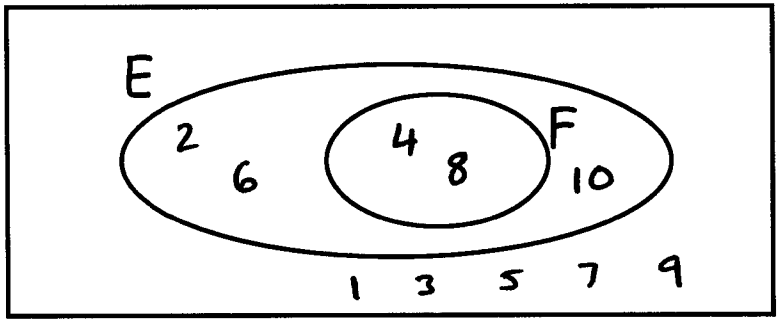


Event C = odd numbers

Event D = numbers in the 3 times table

$P(C) = 5/10 = 1/2$
$P(D) = 3/10$
$p(C \text{ or } D) = p(C \cup D) = 6/10 = 3/5$
$P(C \text{ and } D) = P(C \cap D) = 2/10 = 1/5$
$P(C \text{ given } D) = 2/3$
$P(D \text{ given } C) = 2/5$
$P(\text{not } C) = P(C') = 5/10 = 1/2$

ξ



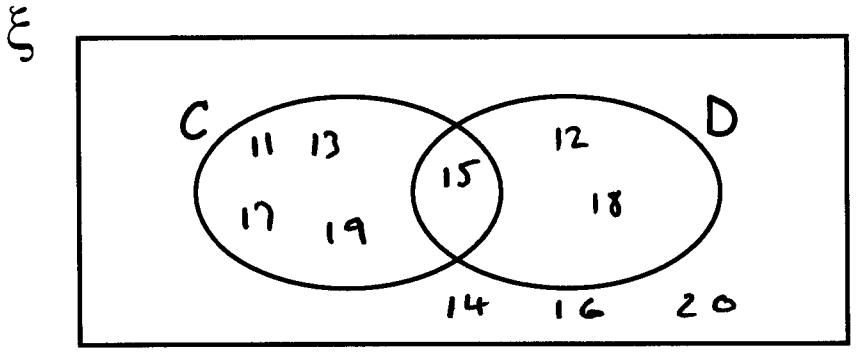
Event E = numbers in the 2 times table

Event F = numbers in the 4 times table

$P(E) = 5/10 = 1/2$
$P(F) = 2/10 = 1/5$
$p(E \text{ or } F) = p(E \cup F) = 5/10 = 1/2$
$P(E \text{ and } F) = P(E \cap F) = 2/10 = 1/5$
$P(E \text{ given } F) = 2/2 = 1$
$P(F \text{ given } E) = 2/5$

VENN DIAGRAMS

Consider the numbers 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20 only (THE UNIVERSAL SET symbol ξ).
Put the numbers in the diagram.

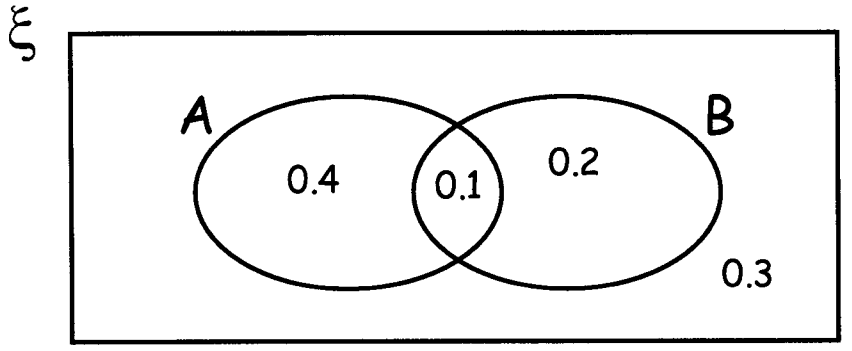


Event C = odd numbers

Event D = numbers in the 3 times table

$P(C) = 5/10 = 1/2$
$P(D) = 3/10$
$p(C \text{ or } D) = p(C \cup D) = 7/10$
$P(C \text{ and } D) = P(C \cap D) = 1/10$
$P(C \text{ given } D) = 1/3$
$P(D \text{ given } C) = 1/5$
$P(\text{not } D) = P(D') = 7/10$

The numbers in this VENN DIAGRAM are PROBABILITIES

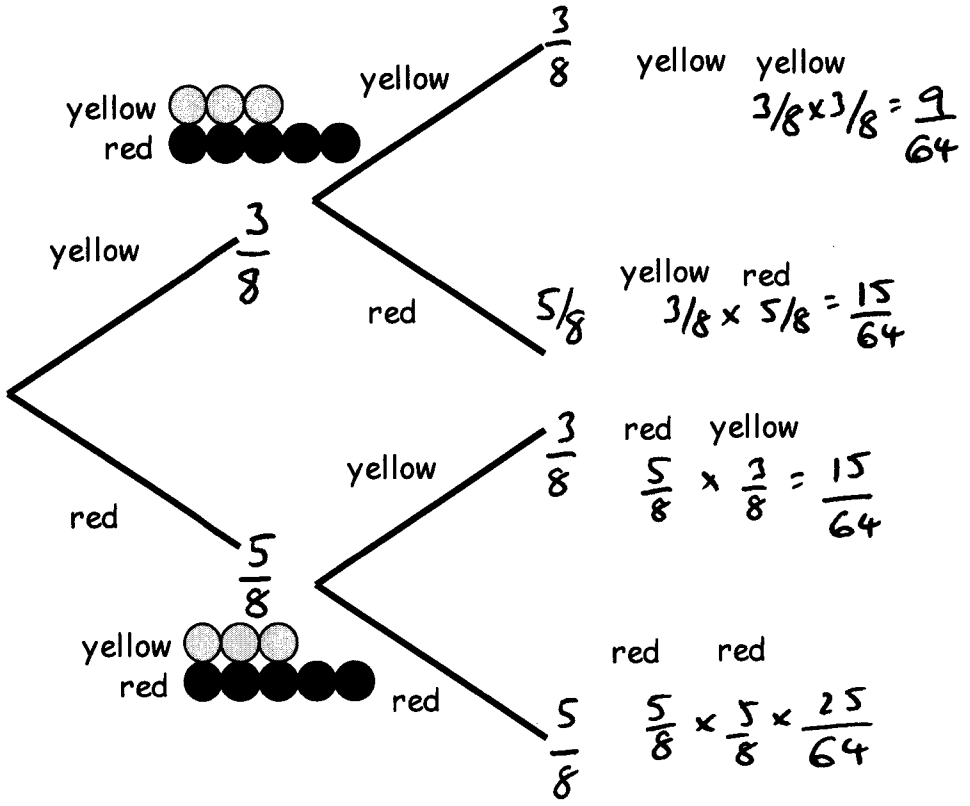


$P(A) = 0.4 + 0.1 = 0.5$
$P(B) = 0.1 + 0.2 = 0.3$
$p(A \text{ or } B) = p(A \cup B) = 0.4 + 0.1 + 0.2 = 0.7$
$P(A \text{ and } B) = P(A \cap B) = 0.1$
$P(A \text{ given } B) = 0.1 \div 0.3 = 0.\dot{3}$
$P(B \text{ given } A) = 0.1 \div 0.5 = 0.2$
$P(\text{not } A) = P(A') = 0.2 + 0.3 = 0.5$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)}$$

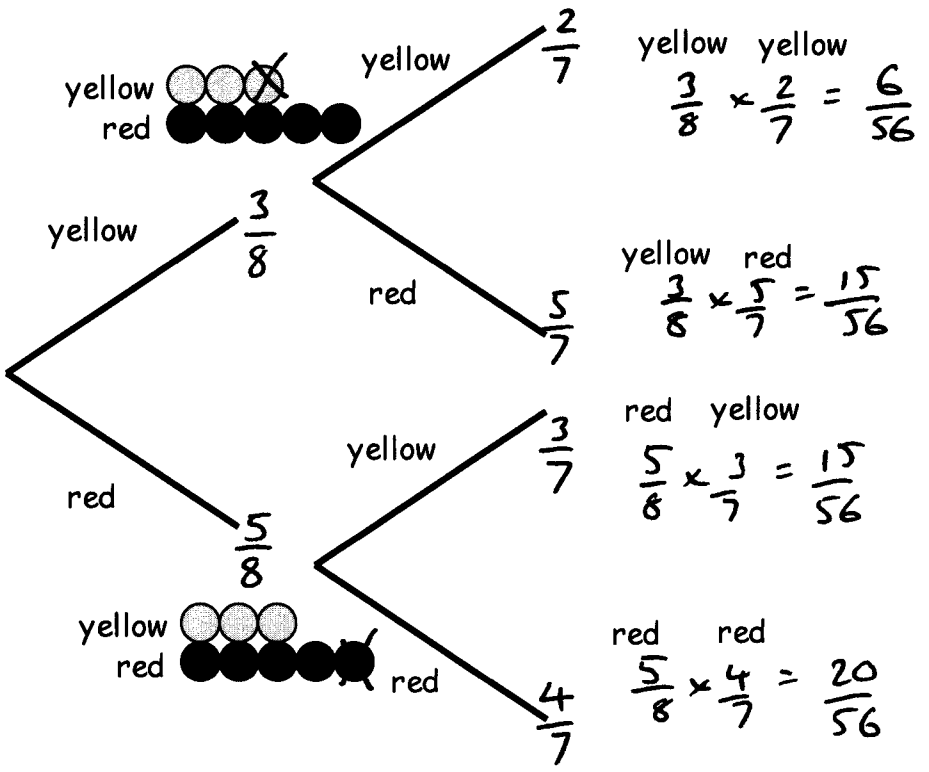
A bag contains 5 red counters and 3 yellow ones.
A counter is chosen and REPLACED. What is the probability of choosing two counters of the same colour?



P(2 same colour) = $\frac{9}{64} + \frac{25}{64} = \frac{34}{64} = \frac{17}{32}$

(YY, RR)

A bag contains 5 red counters and 3 yellow ones.
A counter is chosen and NOT REPLACED. What is the probability of choosing two counters of different colours?

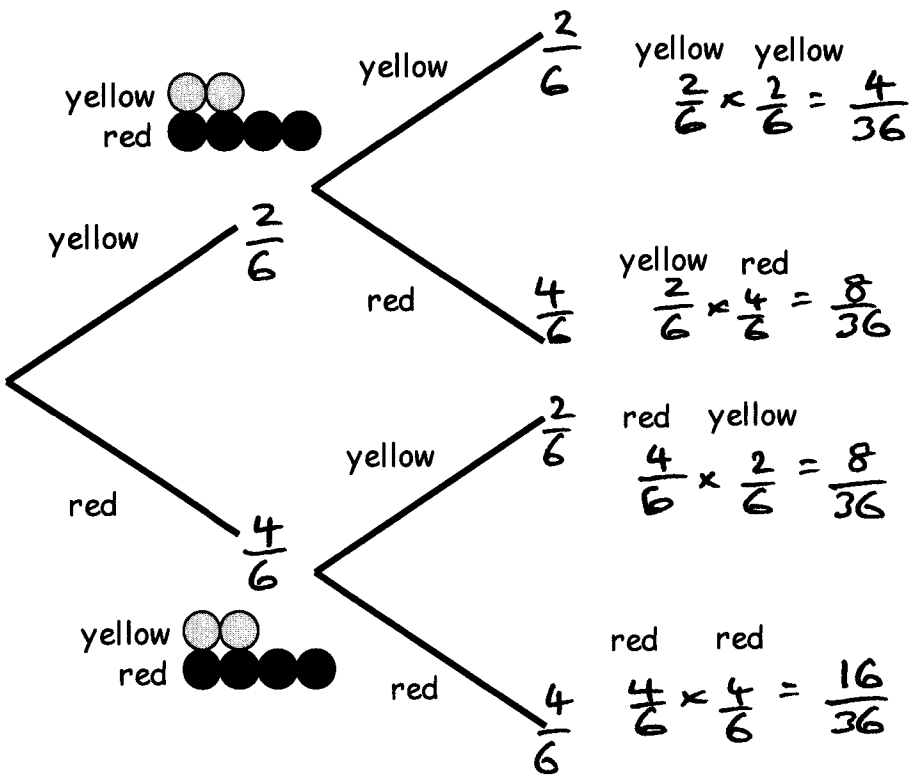


P(2 of different colours)

(YR, RY)

$\frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}$

A bag contains 4 red counters and two yellow ones.
A counter is chosen and REPLACED. What is the probability of choosing two counters of the same colour?

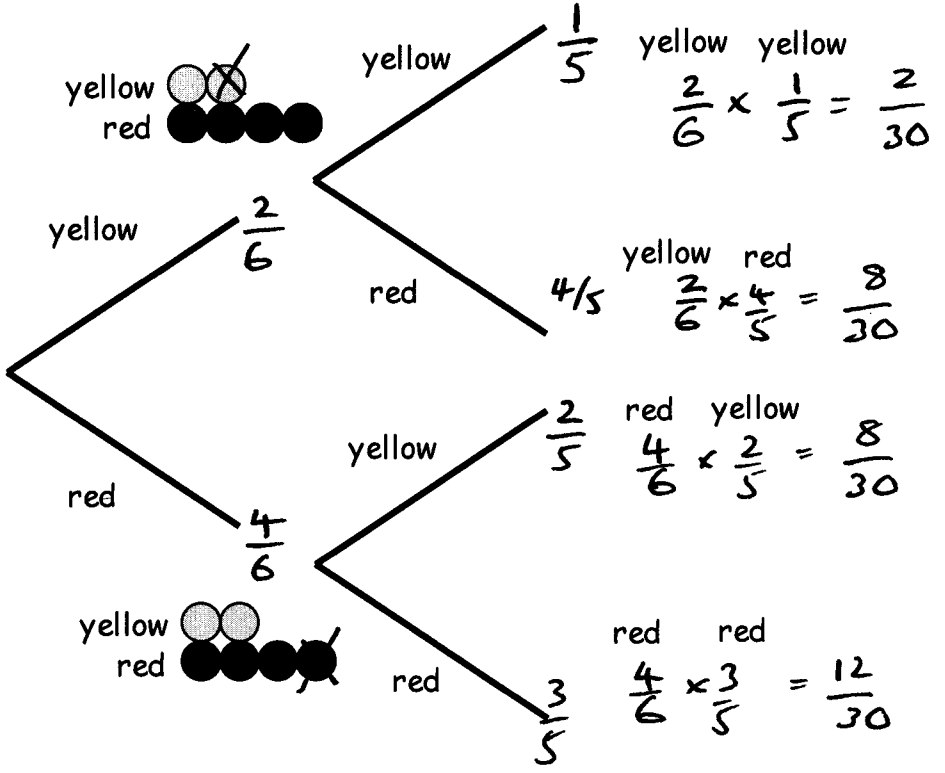


P(2 of the same colour) =

YY + RR

$\frac{4}{36} + \frac{16}{36} = \frac{20}{36} = \frac{5}{9}$

A bag contains 4 red counters and two yellow ones.
A counter is chosen and NOT REPLACED. What is the probability of choosing two counters of the different colours?



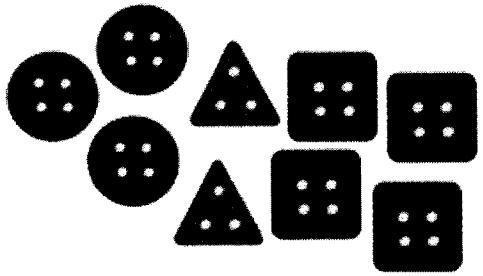
P(2 of different colours) =

YR + RY

$\frac{8}{30} + \frac{8}{30} = \frac{16}{30} = \frac{8}{15}$

Tree diagram WITHOUT replacement

A bag contains 3 round buttons, 2 triangular buttons and 4 square buttons. A button is withdrawn and not replaced, a second button is withdrawn.



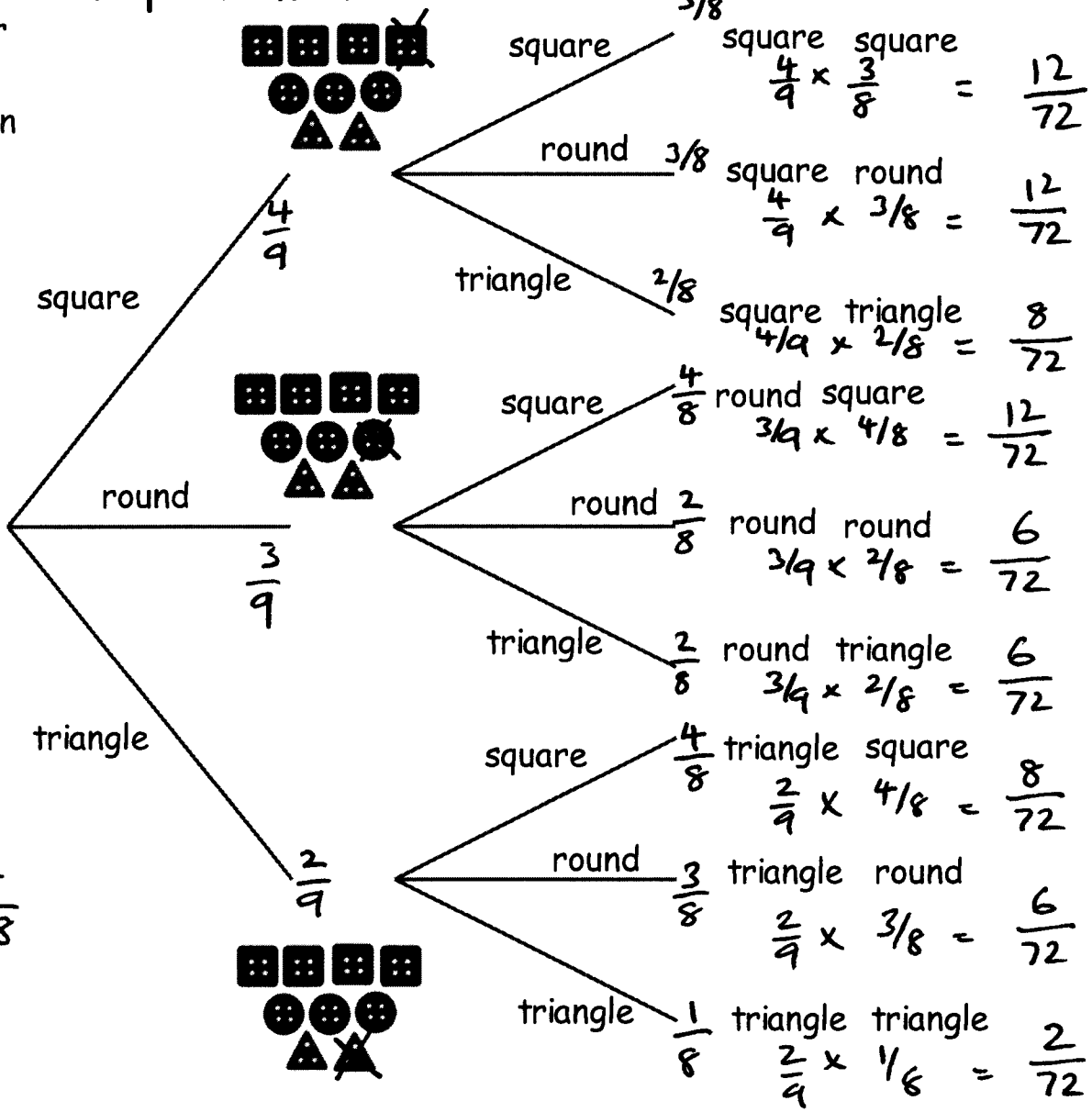
Find the probability of choosing

a) 2 round buttons $\frac{6}{72} = \frac{1}{12}$

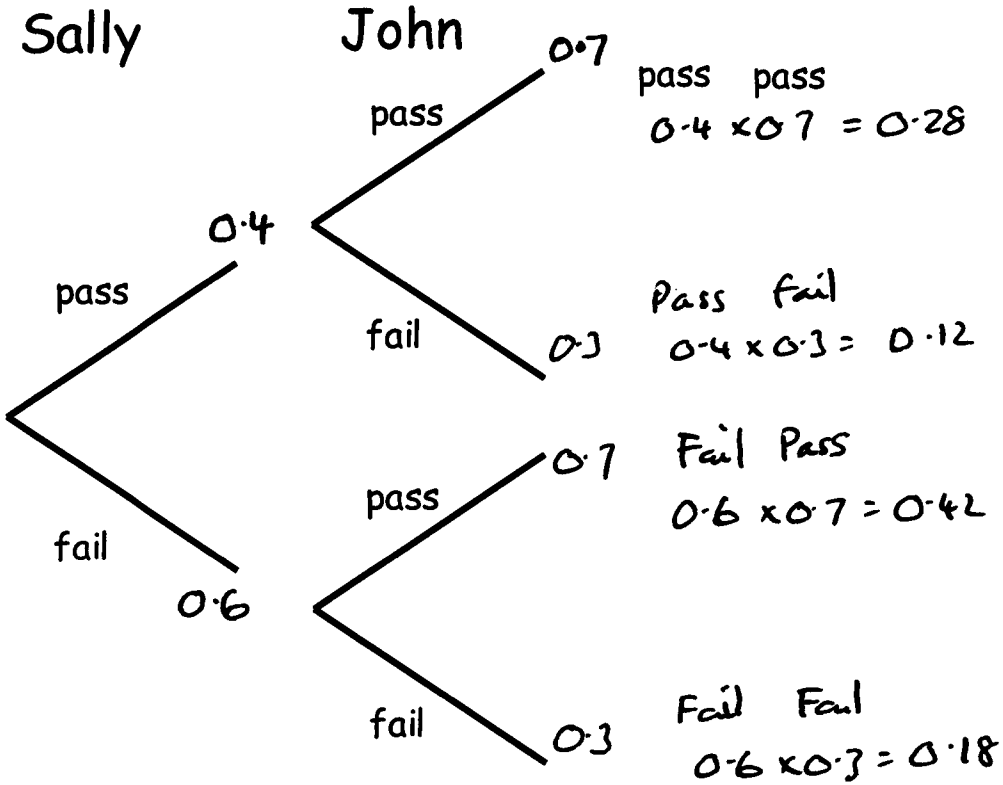
b) 2 buttons that are the same

SS + RR + TT

$\frac{12}{72} + \frac{6}{72} + \frac{2}{72} = \frac{20}{72} = \frac{5}{18}$

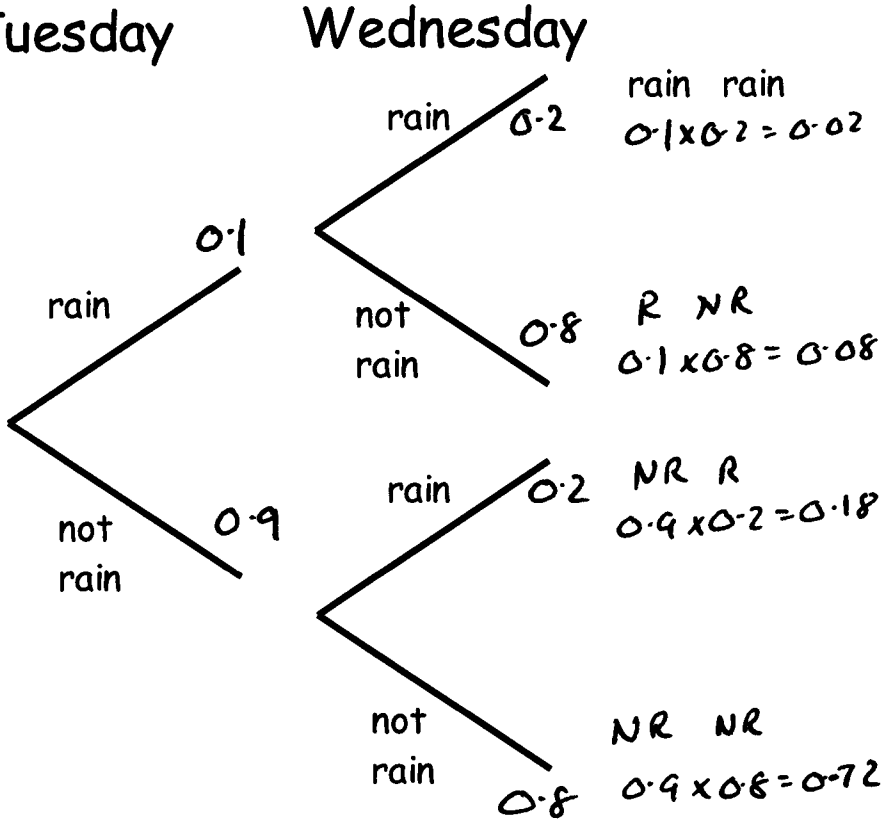


The probability that Sally passes a test is 0.4. The probability that John passes the test is 0.7. What is the probability that at least one of them passes the test?



P(at least one of them Pass) = PP + PF + FP
= 0.28 + 0.12 + 0.42
= 0.82

The probability that it rains on Tuesday is 0.1. The probability that it will rain on Wednesday is 0.2. What is the probability that it will rain on one day only?



P(rain on one day only) =
RNR + NR R
0.08 + 0.18 = 0.26