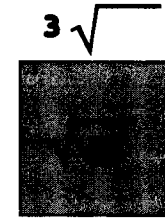
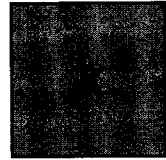


INDICES and STANDARD FORM

Page	Description
1	Square, cube, square root and cube root
2	Square and cube. Power of 0. Square root and cube root as indices
3	Fractional and negative indices. Index laws
4	Simplify expressions involving indices
5	Work out numerical values involving fractional and negative indices
6	Work out numerical values involving fractional and negative indices
7	Mixed examples on all index methods
8	Standard Form
9	Recap on standard form
10	Mixed examples on indices and standard form

Square, square root, cube and cube root

Calculator keys



Work out the following

1) 5^2 25

6) $\sqrt{289}$ 17

2) 8^2 64

7) $\sqrt[3]{2744}$ 14

3) 8^3 512

8) 0.5^2 0.25

4) 10^3 1000

9) 4^3 64

5) $\sqrt{81}$ 9

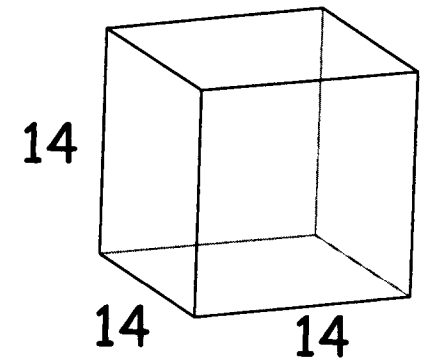
10) $\sqrt[3]{1728}$ 12

Fill in the gaps in these tables

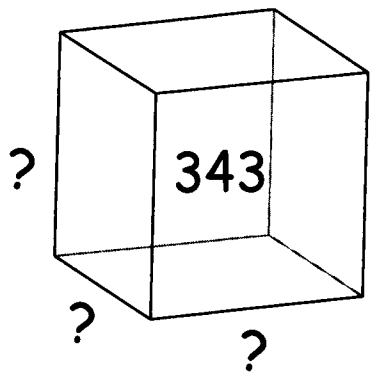
number	number ²
11	121
13	169
19	361
21	441
29	841
18	324

number	number ³
4	64
7	343
11	1331
12	1728
21	9261
18	5832

How many 1 centimetre cubes would it take to make this big cube measuring 14 cm by 14 cm by 14 cm?



$$14 \times 14 \times 14 = 14^3 = 2744$$



It took 343 small 1 cm cubes to make this big cube. What are the lengths of the side of the big cube?

$$\sqrt[3]{343} = 7$$

INDICES

Complete this table

Number	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Cube	8	27	64	125	x	x	x	x	1000	x	x	x	x	x

Square and square root NOTE to square a negative number on your calculator use brackets

$$5^2 = \underline{25} \quad (-5)^2 = \underline{25} \quad \text{so the } \sqrt{25} = \underline{5} \text{ or } \underline{-5}$$

$$6^2 = \underline{36} \quad (-6)^2 = \underline{36} \quad \text{so the } \sqrt{36} = \underline{6} \text{ or } \underline{-6}$$

$$7^2 = \underline{49} \quad (-7)^2 = \underline{49} \quad \text{so the } \sqrt{49} = \underline{7} \text{ or } \underline{-7}$$

Key Fact - Every number greater than 0 has 2 square roots. One positive, one negative

The power of 0

$$3^0 = \underline{1} \quad 5^0 = \underline{1} \quad 9^0 = \underline{1} \quad m^0 = \underline{1}$$

Key Fact - Any number to the power of 0 is 1

Cube Root $2^3 = 8$ so the cube root of 8, written $\sqrt[3]{8} = \underline{2}$

$$\sqrt[3]{1000} = \underline{10} \quad \sqrt[3]{125} = \underline{5} \quad \sqrt[3]{64} = \underline{4}$$

The power of $\frac{1}{2}$

Using your calculator investigate

$$4^{\frac{1}{2}} = \underline{2} \quad 25^{\frac{1}{2}} = \underline{5} \quad 64^{\frac{1}{2}} = \underline{8} \quad 100^{\frac{1}{2}} = \underline{10}$$

Key Fact - The power of $\frac{1}{2}$ is the same as $\sqrt{\quad}$

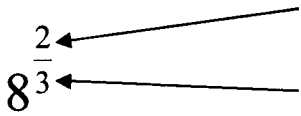
The power of $\frac{1}{3}$

Using your calculator investigate

$$125^{\frac{1}{3}} = \underline{5} \quad 8^{\frac{1}{3}} = \underline{2} \quad 1000^{\frac{1}{3}} = \underline{10} \quad 64^{\frac{1}{3}} = \underline{4}$$

Key Fact - The power of $\frac{1}{3}$ is the same as $\sqrt[3]{\quad}$

Fractional Indices

$$\frac{2}{8^3}$$


Key Fact - for a index that is a fraction,

the numerator = power
the denominator = root

$$4^{\frac{3}{2}} = \underline{8}$$

$$25^{\frac{3}{2}} = \underline{125}$$

$$1000^{\frac{4}{3}} = \underline{10000}$$

$$\sqrt{4} = 2 \quad 2^3 = 8$$

$$\sqrt{25} = 5 \quad 5^3 = 125$$

$$\sqrt[3]{1000} = 10 \quad 10^4 = 10000$$

Negative Index

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$

Key Fact - For a negative index, just write it as 1 over the equivalent positive index

Index Laws

Multiplication - add the index numbers

$$2^3 \times 2^5 = \underline{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2^8 \quad 2^3 \times 2^5 = 2^{3+5} = 2^8$$

Division - subtract the index numbers

$$2^5 \div 2^3 = \frac{\underline{2 \times 2 \times 2 \times 2 \times 2}}{2 \times 2 \times 2} = 2^2 \quad 2^5 \div 2^3 = 2^{5-3} = 2^2$$

To the power of - multiply the index numbers

$$(2^3)^2 = \underline{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2^6 \quad (2^3)^2 = 2^{3 \times 2} = 2^6$$

Further examples

$$1) \quad a^2 \times a^4 = a^{2+4} = a^6 \quad 2) \quad b^7 \div b^3 = b^{7-3} = b^4 \quad 3) \quad (y^3)^4 = y^{3 \times 4} = y^{12}$$

$$4) \quad 3b^2 \times 4b^5 = 12b^7 \quad 5) \quad (a^4)^2 = a^8 \quad 6) \quad \frac{x^7 \times x^2}{x^2 \times x^3} = \frac{x^9}{x^5} = x^4$$

(3)

Simplifying Expressions involving Indices

$$1) a^2 \times a^3 = a^5$$

$$2) a^4 \times a^5 = a^9$$

$$3) 2a^3 \times 4a = 8a^4$$

$$4) 2a^2 \times 3a^3 = 6a^5$$

$$5) (a^4)^2 = a^8$$

$$6) (a^3)^4 = a^{12}$$

$$7) (2a^2)^3 = 8a^6$$

$$8) (3a^4)^2 = 9a^8$$

$$9) \frac{a^5}{a^2} = a^3$$

$$10) \frac{a^7}{a^3} = a^4$$

$$11) \frac{4a^6}{2a^2} = 2a^4$$

$$12) \frac{9a^5}{3a} = 3a^4$$

$$13) \frac{2a^5 \times 6a^3}{3a^2} = \frac{12a^8}{3a^2} = 4a^6$$

$$14) 3a^3b^2 \times 5ab^3 = 15a^4b^5$$

$$15) 4a^2b^5 \times 3a^2b^4 = 12a^4b^9$$

$$16) (3a^3b^2)^2 = 9a^6b^4$$

$$17) (2a^2b^3)^4 = 16a^8b^{12}$$

$$18) \frac{12a^5b^3}{4a^3b^2} = 3a^2b$$

$$19) \frac{9a^7b^4}{6a^5b} = \frac{3}{2}a^2b^3$$

$$20) \frac{8a^4b^2}{6a^4b^3} = \frac{4}{3}b^{-1}$$

Indices

Index	Meaning	Example	Questions		
0	Always = 1	$2^0 = 1$	$4^0 = 1$	$5^0 = 1$	$a^0 = 1$
1	Just the number	$2^1 = 2$	$5^1 = 5$	$8^1 = 8$	$a^1 = a$
-1	1 over the number	$2^{-1} = \frac{1}{2}$	$5^{-1} = \frac{1}{5}$	$8^{-1} = \frac{1}{8}$	$a^{-1} = \frac{1}{a}$
2	Square	$2^2 = 2 \times 2 = 4$	$3^2 = 9$	$4^2 = 16$	$10^2 = 100$
-2	1 over the number squared	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$3^{-2} = \frac{1}{9}$	$4^{-2} = \frac{1}{16}$	$10^{-2} = \frac{1}{100}$
3	Cube	$2^3 = 2 \times 2 \times 2 = 8$	$3^3 = 27$	$4^3 = 64$	$10^3 = 1000$
-3	1 over the number cubed	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	$3^{-3} = \frac{1}{27}$	$4^{-3} = \frac{1}{64}$	$10^{-3} = \frac{1}{1000}$
$\frac{1}{2}$	Square root	$4^{\frac{1}{2}} = \sqrt{4} = 2$	$16^{\frac{1}{2}} = 4$	$49^{\frac{1}{2}} = 7$	$9^{\frac{1}{2}} = 3$
$-\frac{1}{2}$	1 over the square root	$4^{-\frac{1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$	$16^{\frac{1}{2}} = 4$	$49^{-\frac{1}{2}} = \frac{1}{7}$	$9^{-\frac{1}{2}} = \frac{1}{3}$
$\frac{1}{3}$	Cube root	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$	$27^{\frac{1}{3}} = 3$	$64^{\frac{1}{3}} = 4$	$1000^{\frac{1}{3}} = 10$
$-\frac{1}{3}$	1 over the cube root	$8^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$	$27^{-\frac{1}{3}} = \frac{1}{3}$	$64^{-\frac{1}{3}} = \frac{1}{4}$	$1000^{-\frac{1}{3}} = \frac{1}{10}$
$\frac{3}{2}$	Square root then cube	$4^{\frac{3}{2}} = (\sqrt{4})^3 = 8$	$9^{\frac{3}{2}} = 27$	$16^{\frac{3}{2}} = 64$	$25^{\frac{3}{2}} = 125$
$-\frac{3}{2}$	1 over the square root then cube	$4^{-\frac{3}{2}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{8}$	$9^{-\frac{3}{2}} = \frac{1}{27}$	$16^{-\frac{3}{2}} = \frac{1}{64}$	$25^{-\frac{3}{2}} = \frac{1}{125}$
$-\frac{2}{3}$	1 over the cube root then square	$8^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{4}$	$27^{-\frac{2}{3}} = \frac{1}{9}$	$64^{-\frac{2}{3}} = \frac{1}{16}$	$1000^{-\frac{2}{3}} = \frac{1}{100}$

Reminder $3^3 = 27$, $\sqrt[3]{27} = 3$; $4^3 = 64$, $\sqrt[3]{64} = 4$; $10^3 = 1000$, $\sqrt[3]{1000} = 10$

Qu 1) $c) \frac{1 \text{ over } 4 = 1/4$

$$8^{-\frac{2}{3}} = \frac{1}{4}$$

$b) \frac{2^2 = 4}{}$
 $a) \frac{\text{cube root of } 8 = 2}{}$

Qu 4)

$$8^{\frac{1}{3}} = 2$$

$b) \frac{2^1 = 2}{}$
 $a) \frac{\sqrt[3]{8} = 2}{}$

Qu 2) $c) \frac{1}{8}$

$$4^{-\frac{3}{2}} = \frac{1}{8}$$

$b) \frac{2^3 = 8}{}$
 $a) \frac{\sqrt{4} = 2}{}$

Qu 5)

$$100^{\frac{5}{2}} = 100000$$

$b) \frac{10^5 = 100000}{}$
 $a) \frac{\sqrt{100} = 10}{}$

Qu 3) $c) \frac{1}{64}$

$$16^{-\frac{3}{2}} = \frac{1}{64}$$

$b) \frac{4^3 = 64}{}$
 $a) \frac{\sqrt{16} = 4}{}$

Qu 6)

$$9^{\frac{3}{2}} = 27$$

$b) \frac{3^3 = 27}{}$
 $a) \frac{\sqrt{9} = 3}{}$

Indices

Section 1 Workout the numerical answer. E.g. $2^3 = 8$

- | | | | | | |
|----------|----|------------|------|----------------|-----------------|
| 1) 7^2 | 49 | 5) 4^6 | 4096 | 9) 3^{-1} | $\frac{1}{3}$ |
| 2) 2^5 | 32 | 6) 12^0 | 1 | 10) 2^{-3} | $\frac{1}{8}$ |
| 3) 3^4 | 81 | 7) 1^2 | 1 | 11) 10^{-2} | $\frac{1}{100}$ |
| 4) 5^1 | 5 | 8) 110^0 | 1 | 12) 100^{-1} | $\frac{1}{100}$ |

Section 2 Simplify the following, giving your answer in index form.

E.g. $y^2 \times y^3 = y^5$

- | | | |
|-----------------------------|--------------------------------|--|
| 13) $5^6 \times 5^2 = 5^8$ | 17) $(9^2)^5 = 9^{10}$ | 21) $2b^3 \times 4b^4 = 8b^7$ |
| 14) $\frac{4^7}{4^3} = 4^4$ | 18) $\frac{a^4}{a^6} = a^{-2}$ | 22) $\frac{a^4 \times a^3}{a^2} = \frac{a^7}{a^2} = a^5$ |
| 15) $a^2 \times a^7 = a^9$ | 19) $p^{10} \div p^6 = p^4$ | 23) $(3x^2)^3 = 27x^6$ |
| 16) $(b^3)^2 = b^6$ | 20) $2^6 \div 2^4 = 2^2$ | 24) $10b^7 \div 2b^3 = 5b^4$ |

Section 3 Workout the numerical answer. E.g. $4^{\frac{1}{2}} = 2$

- | | | | | | |
|-------------------------|----|-------------------------|---------------|--------------------------|-----------|
| 25) $49^{\frac{1}{2}}$ | 7 | 29) $4^{\frac{3}{2}}$ | 8 | 33) $32^{\frac{2}{5}}$ | 4 |
| 26) $121^{\frac{1}{2}}$ | 11 | 30) $9^{-\frac{1}{2}}$ | $\frac{1}{3}$ | 34) $(-8)^{\frac{1}{3}}$ | -2 |
| 27) $64^{\frac{1}{3}}$ | 4 | 31) $27^{-\frac{1}{3}}$ | $\frac{1}{3}$ | 35) $(-4)^{\frac{1}{2}}$ | cannot do |
| 28) $8^{\frac{2}{3}}$ | 4 | 32) $16^{\frac{3}{4}}$ | 8 | 36) $100^{\frac{3}{2}}$ | 1000 |

STANDARD FORM KEY SKILLS

1) Which number is not in standard form?

4.3×10^4 43×10^4 4.3×10^{-2}

43 not between 1 and 10

2) Write these numbers in standard form

a) 45600 b) 8000 c) 0.0206
 4.56×10^4 8×10^3 2.06×10^{-2}

3) Write these numbers in normal form

a) 2.09×10^4 b) 1.7×10^3 c) 7.956×10^{-2}
 20900 1700 0.07956

4) Write these numbers in order of size, smallest to biggest

3×10^{-2} 3×10^4 2.7×10^{-2} 2.7×10^{-2}
 3×10^{-2} 3×10^4 3×10^4

5) Without using a calculator work out the following.

Give your answer in standard form.

a) $2 \times 10^4 \times 3 \times 10^3$ c) $4 \times 10^4 \div 2 \times 10^2$

b) $4 \times 10^5 \times 3 \times 10^3$ d) $4 \times 10^5 \div 8 \times 10^3$

a) 6×10^7
 b) 12×10^8
 $= 1.2 \times 10^9$

c) 2×10^2
 d) 0.5×10^2
 $= 5 \times 10^1$

6) Using a calculator work out the following.

Give your answer in standard form.

Remember use the  button

a) $4.3 \times 10^{-2} \times 7.8 \times 10^8 = 33540000$
 $= 3.354 \times 10^7$

b) $4 \times 10^9 \times 4 \times 10^6 = 1.6 \times 10^{16}$

c) $6 \times 10^9 \div 1.2 \times 10^{-2} = 5 \times 10^{11}$

d) $(6 \times 10^3)^2 = 36000000$
 $= 3.6 \times 10^7$

e) $4 \times 10^6 + 4 \times 10^5 = 4400000$
 $= 4.4 \times 10^6$

f) $1.3 \times 10^5 - 4 \times 10^3$
 $= 126000$
 $= 1.26 \times 10^5$

Standard Form

Write these numbers in standard form

1) 0.0000365 3.65×10^{-5}

2) 7300 7.3×10^3

3) 210 2.1×10^2

4) 0.0000201 2.01×10^{-5}

Write these numbers in normal form

5) 2.7×10^4 27000

6) 1.6×10^{-3} 0.0016

7) 4.07×10^5 407000

8) 2.06×10^{-4} 0.000206

9) Write these numbers in order of size smallest to largest.

2.07×10^5 , 2.3×10^{-5} , 2.1×10^5 , 2.07×10^4

2.3×10^{-5} , 2.07×10^4 , 2.07×10^5 , 2.1×10^5

Without a calculator work out the answers

10) $3 \times 10^4 \times 4 \times 10^3 = 12 \times 10^7 = 1.2 \times 10^8$

11) $\frac{8 \times 10^5}{2 \times 10^2} = 4 \times 10^3$

With a calculator work out the answers.

Give your answers in standard form to 1dp

12) $(2.07 \times 10^7)^2 = 4.2849 \times 10^{14} = 4.3 \times 10^{14}$ to 1dp

13) $\frac{2.3 \times 10^5 + 5.7 \times 10^6}{2.4 \times 10^{-2} - 1.07 \times 10^{-3}} = 258613170.5 = 2.6 \times 10^8$

9

Revision on Indices and Standard Form

1) $3^6 \times 3^7 = 3^? \quad 3^{13}$ 1 mark

2) $\frac{3^7}{3^2} = 3^? \quad 3^5$ 1 mark

3) $(3^2)^4 = 3^? \quad 3^8$ 1 mark

4) Simplify $5c^4d^2 \times c^2d^3$ 2 marks
 $5c^6d^5$

5) Simplify $(3x^2y^4)^2$ 2 marks
 $9x^4y^8$

6) Simplify $(2x^5y^4z^6) \times (7x^2y^3z)$ 3 marks
 $14x^7y^7z^7$

7) Solve $x^2 = 36$ 2 marks
 $x = \sqrt{36}$
 $x = 6 \text{ or } -6$

8) Simplify $(2a^3b)^4$ 2 marks
 $16a^{12}b^4$

9) Write these in order of size. Smallest first

$27^{\frac{2}{3}}$, $64^{\frac{1}{3}}$, $4^{\frac{3}{2}}$
 9 , 4 , 8
 $64^{\frac{1}{3}}$, $4^{\frac{3}{2}}$, $27^{\frac{2}{3}}$

10) $(2a^2)^3$
 $8a^6$

Write these numbers in standard form

11) 0.0025 2.5×10^{-3}

12) 630 6.3×10^2

13) 5003 5.003×10^3

14) 0.00002 2×10^{-5}

Write these numbers in normal form

15) 5.7×10^3 5700

16) 2.6×10^{-2} 0.026

17) 2.07×10^5 207000

18) 1.06×10^{-4} 0.000106

19) Write these numbers in order of size smallest to largest.

2.07×10^5 , 2.3×10^{-5} , 2.1×10^5 , 2.07×10^4
 2.3×10^{-5} , 2.07×10^4 , 2.07×10^5 , 2.1×10^5

Without a calculator work out the answers

21) $3 \times 10^4 \times 4 \times 10^3 = 1.2 \times 10^8$

22) $\frac{8 \times 10^5}{2 \times 10^2} = 4 \times 10^3$

With a calculator work out the answers.

Give your answers in standard form to 1dp

23) $(2.07 \times 10^7)^2 = 4.3 \times 10^{14}$

24) $\frac{2.3 \times 10^5 + 5.7 \times 10^6}{2.4 \times 10^{-2} - 1.07 \times 10^{-3}} = 2.6 \times 10^8$