

ALGEBRA - Substitute, Simplify, expand, factorise

Page	Description
1	Substitute
2	Simplify and writing expressions using algebra
3	Simplify, expand single bracket, factorise single bracket
4	Simplify, expand single bracket, expand and simplify, factorise single bracket
5	Expand two brackets. Factorise quadratics
6	Expand single bracket, expand and simplify, expand two brackets, factorise quadratics, factorise single bracket
7	Expand and factorise single bracket and two brackets
8	Simplify expressions. Expand single bracket, expand and simplify, expand two brackets, factorise quadratics, factorise single bracket
9	Recap on factorising
10	Factorise harder quadratics
11	Factorise harder quadratics. Simplify algebraic fractions
12	Simplify algebraic fractions

Substitution

For questions 13 to 24

1) $a = 3$ $\boxed{a} + 3 = 6$

$a = 4, b = 3 \text{ and } c = 2$

2) $b = 7$ $\boxed{b} - 1 = 6$

13) $a + 9 = 4 + 9 = 13$

3) $e = 4$ $3e$
 $3 \times \boxed{4} = 12$

15) $3c + a = 3 \times 2 + 4 = 6 + 4 = 10$

4) $s = 8$ $\boxed{s} \div 2 = 4$

17) $2(a + b) = 2(4 + 3) = 2 \times 7 = 14$

5) $p = 6$ $2 \times \boxed{p} + 1 = 13$

19) $13 - a = 13 - 4 = 9$

6) $q = 7$ $3 \times \boxed{q} - 4 = 17$

20) $ab + c = 4 \times 3 + 2 = 12 + 2 = 14$

7) $r = 8$ $3 \times (r - 5) = 9$

21) $4(a - c) = 4(4 - 2) = 4 \times 2 = 8$

8) $a = 2$
 $b = 3$ $\boxed{a} + \boxed{b} = 5$

22) $4c + 7 = 4 \times 2 + 7 = 8 + 7 = 15$

9) $a = 2$
 $b = 3$ $\boxed{a} \times \boxed{b} = 6$

23) $20 - 2a = 20 - 2 \times 4 = 20 - 8 = 12$

10) $c = 4$
 $d = 5$ $4 \times \boxed{c} + \boxed{d} = 16 + 5 = 21$

24) $abc = 4 \times 3 \times 2 = 24$

11) $c = 2$
 $d = 5$ $4 \times \boxed{c} \times \boxed{d} = 40$

12) $e = 6$
 $f = 2$ $3 \times \boxed{e} + 4 \times \boxed{f} = 18 + 8 = 26$

(1)

Simplify where possible

1) $a + a$ $2a$

2) $2a - a$ a

3) $3 \times 2a$ $6a$

4) $a \times a$ a^2

5) $2a + a$ $3a$

6) $\frac{2a}{a}$ 2

7) $a - a$ 0

8) $a^2 + a^2 + a$ $2a^2 + a$

9) $a^2 - a$ $a^2 - a$

10) $a^2 \times a$ a^3

11) $2a + 3b - 5a + b + 5$

$$-3a + 4b + 5$$

12) $2s \times 3s$ $6s^2$

13) $y^2 \times y^3$ y^5

14) $y^2 + y^3$ $y^2 + y^3$

15) $3ab \times 2a$

$$6a^2b$$

Writing in algebra

1) What number is two more than n?

$$n + 2$$

2) What number is three less than n?

$$n - 3$$

3) What number is twice as big as n?

$$2n$$

4) What number is a quarter the size of n?

$$\frac{n}{4}$$

5) Write in terms of n an expression that is always odd.

$$2n + 1$$

 6) Show that the expression $4n - 1 + n + 11$ is always a multiple of 5.

$$4n - 1 + n + 11$$

$$= 5n + 10$$

$$= 5(n + 2)$$

this is a
multiple of 5

(2)

Write in a more simple way

1) $x + x + x + x = 4x$

2) $y \times y = y^2$

3) 3 lots of $2x = 6x$

4) 2 lots of $3y = 6y$

5) $x^2 + x = x^2 + x$

6) $4 \times 2y = 8y$

Simplify these ones

1) $3a + 4a + a = 8a$

2) $7x - 4x = 3x$

3) $a + 2b + b + 4a = 5a + 3b$

4) $5p + q - 2q = 5p - q$

5) $4g + 5 + g - 2 = 5g + 3$

6) $4a + 3a - 7a = 0$

7) $4g + 2h + 3 + g - 3h = 5g - h + 3$

8) $9b + 2c - 3b = 6b + 2c$

9) $x + x + s + 3s + 2x = 4x + 4s$

10) $2y + h - 3y - 2h = -y - h$

Expand these brackets

1) $2(x + 3) = 2x + 6$

2) $4(y + 3) = 4y + 12$

3) $3(2a + 1) = 6a + 3$

4) $2(5y + 3) = 10y + 6$

5) $6(2x + 1) = 12x + 6$

6) $4(3x - 1) = 12x - 4$

7) $2(5x - 1) = 10x - 2$

8) $3(2 + 3y) = 6 + 9y$

Factorise these expressions

1) $2x + 4 = 2(\underline{x} + \underline{2})$

2) $2x + 6 = 2(\underline{x} + \underline{3})$

3) $3x + 3 = 3(\underline{x} + \underline{1})$

4) $3x + 6 = 3(\underline{x} + \underline{2})$

5) $4x + 8 = 4(\underline{x} + \underline{2})$

6) $3y + 9 = 3(\underline{y} + \underline{3})$

7) $4x + 6 = 2(\underline{2x} + \underline{3})$

8) $9y + 6 = 3(\underline{3y} + \underline{2})$

Algebra Review

Simplify the following expressions if possible

1) $a + a + a = \underline{3a}$

2) $a \times a = \underline{a^2}$

3) $2a \times a = \underline{2a^2}$

4) $a^2 + 2a^2 = \underline{3a^2}$

5) $a^2 + 2a = \underline{a^2 + 2a}$

6) $5\pi + 2\pi = \underline{7\pi}$

7) $5a - 6a = \underline{-a}$

8) $2a + 3b = \underline{2a + 3b}$

9) $a \times a^2 = \underline{a^3}$

10) $4a \div 2 = \underline{2a}$

11) $4a \div 2a = \underline{2}$

12) $2a \times 3a \times 4 = \underline{24a^2}$

Work out the answers to

13) $-4 - 3 = \underline{-7}$

14) $-2 + 7 = \underline{5}$

15) $6 - 8 = \underline{-2}$

16) $3 \times -2 = \underline{-6}$

17) $-4 \times -3 = \underline{12}$

18) $-8 \times 2 = \underline{-16}$

19) $-3 \times 2 + -4 \times -2 = \underline{2}$
 $\quad \quad \quad -6 + 8$

20) $-3 + 5 - 8 - 4 + 2 = \underline{-8}$

Expand

21) $3(2x + 3) = \underline{6x + 9}$

22) $x(x + 1) = \underline{x^2 + x}$

23) $4x(3 - 2x) = \underline{12x - 8x^2}$

24) $-2(3 - 2x) = \underline{-6 + 4x}$

Expand and simplify

25) $3(x - 3) + 2(5 - 2x) = \underline{3x - 9 + 10 - 4x}$
 $\quad \quad \quad = \underline{-x + 1}$

Expand

Simplify

26) $4(2x - 1) - 2(3x - 2) = \underline{8x - 4 - 6x + 4}$
 $\quad \quad \quad = \underline{2x}$

Expand

Simplify

27) $7 - (4 - x) = \underline{7 - 4 + x}$
 $\quad \quad \quad = \underline{3 + x}$

Expand

Simplify

28) $x(x - 2) - 3x(2x + 1) = \underline{x^2 - 2x - 6x^2 - 3x}$
 $\quad \quad \quad = \underline{-5x^2 - 5x}$

Expand

Simplify

Factorise

29) $4x + 8 = \underline{4(x + 2)}$

30) $3x^2 - 7x = \underline{x(3x - 7)}$

31) $9x^2 - 6x = \underline{3x(3x - 2)}$

32) $4ab^2 - 6a^2b = \underline{2ab(2b - 3a)}$

Expanding and Factorising Quadratics

Expand and simplify the following

$$1) \quad (\overbrace{x+2}^{\text{1}})(\overbrace{x+3}^{\text{2}}) = x^2 + 3x + 2x + 6 \\ = x^2 + 5x + 6$$

$x \times x + 2 \times 3 + 2 \times x + 2 \times 3$

$$2) \quad (x-4)(x+3) \\ x^2 - x - 12$$

$$3) \quad (x+5)(x-2) \\ x^2 + 3x - 10$$

$$4) \quad (x-2)(x-7) \\ x^2 - 9x + 14$$

Find two numbers that

Qu.	Times to make	Add to make	Answers	Qu.	Times to make	Add to make	Answers
9)	6	5	2 3	14)	-12	-4	-6 2
10)	8	6	2 4	15)	-24	2	6 -4
11)	-8	2	4 -2	16)	10	-11	-1 -10
12)	-6	1	3 -2	17)	-16	0	-4 4
13)	8	-6	-2 -4	18)	0	4	0 4

Factorise these quadratics

$$19) \quad x^2 + 6x + 8 = (x+2)(x+4)$$

2 numbers times to make 8
2 and 4
Add to make 6

$$20) \quad x^2 + 10x + 21 = (x+3)(x+7)$$

2 numbers times to make 21
3 and 7
Add to make 10

$$21) \quad x^2 - x - 6 = (x-3)(x+2)$$

2 numbers times to make -6
-3 and 2
Add to make -1

$$22) \quad x^2 - 2x - 15 = (x-5)(x+3)$$

2 numbers times to make -15
-5 and 3
Add to make -2

$$23) \quad x^2 - 8x + 15 = (x-3)(x-5)$$

times 15
add -8 -3 and -5

$$24) \quad x^2 - 25 = (x-5)(x+5)$$

$x \rightarrow -25$
 $+ \rightarrow 0$ -5 and 5

$$25) \quad x^2 - x - 20 = (x-5)(x+4)$$

$x \rightarrow -20$
 $+ \rightarrow -1$ -5 and 4

$$26) \quad x^2 - 10x + 16 = (x-2)(x-8)$$

$x \rightarrow 16$
 $+ \rightarrow -10$ -2 and -8

$$27) \quad x^2 + 5x - 24 = (x+8)(x-3)$$

$x \rightarrow 24$
 $+ \rightarrow 5$ 8 and -3

$$28) \quad x^2 - 2x - 35 = (x-7)(x+5)$$

$x \rightarrow -35$
 $+ \rightarrow -2$ -7 and 5

$$29) \quad x^2 + 8x + 7 = (x+1)(x+7)$$

$x \rightarrow 7$
 $+ \rightarrow 8$ 1 and 7

$$30) \quad x^2 - 100 = (x-10)(x+10)$$

$x \rightarrow -100$
 $+ \rightarrow 0$ -10 and 10

Algebra Revision

Expand

$$1) \quad 3(2x + 3) = \underline{6x + 9} \quad 2) \quad x(x + 1) = \underline{x^2 + x}$$

$$3) \quad 4x(3 - 2x) = \underline{12x - 8x^2}$$

Expand and simplify

$$4) \quad 3(x - 3) + 2(5 - 2x) = \underline{3x - 9 + 10 - 4x} \quad \text{Expand}$$

$$= \underline{-x + 1} \quad \text{Simplify}$$

$$5) \quad 4(2x - 1) - 2(3x - 2) = \underline{8x - 4 - 6x + 2} \quad \text{Expand}$$

$$= \underline{2x - 2} \quad \text{Simplify}$$

$$6) \quad 7 - 2(4 - x) = \underline{7 - 8 + 2x} \quad \text{Expand}$$

$$= \underline{-1 + 2x} \quad \text{Simplify}$$

Expand

$$7) \quad (x + 1)(x + 3)$$

times	x	+1
x	x^2	x
+3	$3x$	3

$$\underline{x^2 + 4x + 3} \quad \text{simplify}$$

$$8) \quad (x - 1)^2$$

times	x	-1
x	x^2	-x
-1	-x	+1

$$\underline{x^2 - 2x + 1} \quad \text{simplify}$$

this means $(x - 1)(x - 1)$

$$9) \quad (x - 2)(2x + 3)$$

times	x	-2
2x	$2x^2$	-4x
+3	$3x$	-6

$$\underline{2x^2 - x - 6} \quad \text{simplify}$$

Factorise

$$10) \quad x^2 + 3x + 2 = (x + \underline{2})(x + \underline{1}) \quad \text{two numbers, times make 2, add make 3}$$

$$11) \quad x^2 + 7x + 6 = (x + \underline{6})(x + \underline{1}) \quad \text{two numbers, times make 6, add make 7}$$

$$12) \quad x^2 + 7x + 12 = (x + \underline{3})(x + \underline{4}) \quad \text{two numbers, times make 7, add make 12}$$

Factorise

$$13) \quad 4x + 8 = 4(\underline{x} + \underline{2})$$

$$14) \quad 2x^2 - x = x(\underline{2x} - \underline{1})$$

$$15) \quad 9x^2 - 6x = 3x(\underline{3x} - \underline{2})$$

Multiplying out or expanding

Single bracket

$$2(x - 4) = 2x - 8$$

$$x(x + 2) = x^2 + 2x$$

$$2x(3x - 1) = 6x^2 - 2x$$

Two brackets

$$(x + 2)(x + 4) = x^2 + 6x + 8$$

$$(x - 1)(x + 4) = x^2 + 3x - 4$$

$$(x - 2)(x - 5) = x^2 - 7x + 10$$

What type of factorising

Single bracket

$$3x + 6 = 3(x + 2)$$

$$x^2 - 4x = x(x - 4)$$

$$4x^2 - 8x = 4x(x - 2)$$

$$\frac{6a^2b - 4ab^2}{2ab} = 3a - 2b$$

Two brackets

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

$$x^2 - x - 6 = (x - 3)(x + 2)$$

$$x^2 + x - 6 = (x + 3)(x - 2)$$

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$

$$x^2 - 36 = (x - 6)(x + 6)$$

Multiply out (Expand)

$$1) x(2x - 3) = 2x^2 - 3x$$

$$2) 7(x - 2) = 7x - 14$$

$$3) 3x(2 - x) = 6x - 3x^2$$

$$4) (x - 2)(x + 7) = x^2 + 5x - 14$$

$$5) (x - 4)(x - 5) = x^2 - 9x + 20$$

$$6) (x + 2)(2 - x) = 4 - x^2$$

$$7) (x + 5)(x - 5) = x^2 - 25$$

$$8) (2x - 3)(3x - 2) = 6x^2 - 13x + 6$$

Factorise

$$1) 4x + 2 = 2(2x + 1)$$

$$2) 6x^2 - x = x(6x - 1)$$

$$3) 6x^2 - 2x = 2x(3x - 1)$$

$$4) x^2 - 16 = (x - 4)(x + 4)$$

$$5) x^2 + 5x + 4 = (x + 4)(x + 1)$$

$$6) x^2 - 5x + 4 = (x - 1)(x - 4)$$

$$7) x^2 - 3x - 4 = (x - 4)(x + 1)$$

$$8) x^2 + 3x - 4 = (x + 4)(x - 1)$$

Expressions

Add and take

$$3x + 2y - 5 + 6x - 4y - 2 = 8x - 2y - 7$$

$$4a^2 + 5a^2 = 9a^2$$

$$9x^2y^3 - 6x^2y^3 + 2x^3y^2 = 3x^2y^3 + 2x^3y^2$$

Multiply

$$5 \times 3a = 15a$$

$$5x \times 2x = 10x^2$$

$$4a^2 \times 3a^3 = 12a^5$$

Divide

$$6x \div 2 = 3x$$

$$8y^3 \div 2y = 4y^2$$

$$\frac{12x^2y^3}{3xy^2} = 4xy$$

Expand single brackets

$$3(x - 3) = 3x - 9$$

$$y(y + 2) = y^2 + 2y$$

$$2x(3x + 4) = 6x^2 + 8x$$

Expand two brackets

$$(x - 3)(x + 7) = x^2 + 4x - 21$$

$$(x + 1)(x + 6) = x^2 + 7x + 6$$

$$(x - 3)(x - 4) = x^2 - 7x + 12$$

$$(x - 2)^2 = x^2 - 4x + 4$$

Expand and simplify

$$2(3x + 5) + 4(x - 3) = 6x + 10 + 4x - 12 = 10x - 2$$

$$3(x - 2) - 2(x - 4) = 3x - 6 - 2x + 8 = x + 2$$

$$3x - (x - 4)$$

$$3x - x + 4 = 2x + 4$$

Factorise into 1 bracket

$$2x + 6 = 2(x + 3)$$

$$x^2 - 2x = x(x - 2)$$

$$4x^2 + 6x = 2x(2x + 3)$$

$$10a^2b^3 + 15a^3b^2 = 5a^2b^2(2b + 3a)$$

Factorise into 2 brackets

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

$$x^2 - x - 12 = (x - 4)(x + 3)$$

$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

$$x^2 - 6x + 5 = (x - 5)(x - 1)$$

$$x^2 - 25 = (x - 5)(x + 5)$$

1) 1 bracket

Clue: there is one number factor and no algebra factor. The factor is 3

Answer $3(x + 4)$

2) 2 brackets

Clue: Find two numbers times to make -12 and add to make -1. 3 and -4

Answer $(x + 3)(x - 4)$

3) 1 bracket

Clue: there are no number factors and one algebra factor. The factor is x Answer $x(x + 2)$

4) 1 bracket

Clue: there is one number factors and one algebra factor. The factors are $2x$ Answer $2x(2x + 5)$

5) 2 brackets

Clue: Find two numbers times to make 12 and add to make 7. 3 and 4

Answer $(x + 3)(x + 4)$

6) 2 brackets

Clue: Find two numbers times to make -8 and add to make 2. -2 and 4

Answer $(x - 2)(x + 4)$

7) 2 brackets

Clue: Find two numbers times to make 6 and add to make -5. -2 and -3

Answer $(x - 2)(x - 3)$

8) 1 bracket

Clue: there are no number factors and one algebra factor. The factor is x^2 Answer $x^2(x + 3)$

9) 2 brackets

Clue: Find two numbers times to make -16 and add to make 0. 4 and -4

Answer $(x + 4)(x - 4)$

10) 1 bracket

Clue: there is one number factors and one algebra factor. The factors are $2x^2$ Answer $2x^2(3 - 8x)$

	x^2 term x term number term	$2x^2 + 5x + 3$	$2x^2 + 3x - 9$	$3x^2 - 5x - 2$
Multiply x^2 term and number term together		$2 \times 3 = 6$	$2x - 9 = -18$	$3x^2 = -6$
Find two numbers that multiply together make this new number but add to make the x term	times = 6 add = 5	2 and 3	$x \rightarrow -18$ $+ \rightarrow 3$ 6 and -3	$x \rightarrow -6$ $+ \rightarrow -5$ -6 and 1
Split the x term using these two numbers		$5x = 2x + 3x$	$3x = 6x - 3x$	$-5x = -6x + x$
Re-write the equation using this split		$2x^2 + 2x + 3x + 3$	$2x^2 + 6x - 3x - 9$	$3x^2 - 6x + x - 2$
Factorise the front two pairs and the back two pairs creating the same term in the brackets		$2x(x + 1) + 3(x + 1)$	$2x(x + 3) - 3(x + 3)$	$3x(x - 2) + 1(x - 2)$
Factorise again to give the answer		$(x + 1)(2x + 3)$	$(x + 3)(2x - 3)$	$(x - 2)(3x + 1)$

$$1) 6x^2 + 5x + 1 = (3x + 1)(2x + 1)$$

$$2) 2x^2 + 11x + 12 = (x + 4)(2x + 3)$$

$$3) 2x^2 - x - 6 = (2x + 3)(x - 2)$$

$$4) 4x^2 - 7x - 2 = (4x + 1)(x - 2)$$

$$5) 6x^2 - 5x + 1 = (3x - 1)(2x - 1)$$

$$6) 4x^2 - 8x + 3 = (2x - 3)(2x - 1)$$

$$7) x^2 - 16 = (x - 4)(x + 4)$$

$$8) x^2 - 4 = (x - 2)(x + 2)$$

Simplify

$$\text{Qu. 1} \quad \frac{2x^2 + 11x + 12}{2x^2 - x - 6} = \frac{(x+4)(2x+3)}{(2x+3)(x-2)} = \frac{x+4}{x-2}$$

$$\text{Qu. 2} \quad \frac{2x^2 - x - 6}{4x^2 - 7x - 2} = \frac{(2x+3)(x-2)}{(4x+1)(x-2)} = \frac{2x+3}{4x+1}$$

$$\text{Qu. 3} \quad \frac{6x^2 - 5x + 1}{4x^2 - 8x + 3} = \frac{(3x-1)(2x-1)}{(2x-3)(2x-1)} = \frac{3x-1}{2x-3}$$

$$\text{Qu. 4} \quad \frac{2x^2 + 11x + 12}{x^2 - 16} = \frac{(x+4)(2x+3)}{(x-4)(x+4)} = \frac{2x+3}{x-4}$$

$$\text{Qu. 5} \quad \frac{2x^2 - x - 6}{x^2 - 4} = \frac{(2x+3)(x-2)}{(x-2)(x+2)} = \frac{2x+3}{x+2}$$

$$\text{Qu. 6} \quad \frac{9x^2 - 4}{4x^2 - 8x + 3} = \frac{(3x-2)(3x+2)}{(2x-3)(2x-1)}$$

cannot cancel -

Algebraic Fractions

1)
$$\frac{1}{x} + \frac{1}{y}$$

$$= \frac{1 \times y + 1 \times x}{x \times y}$$

$$= \frac{y + x}{xy}$$

2)
$$\frac{a}{x} + \frac{b}{y}$$

$$\frac{ay + bx}{xy}$$

3)
$$\frac{2}{x} - \frac{3}{y}$$

$$\frac{2y - 3x}{xy}$$

4)
$$\frac{a}{x} \times \frac{b}{y}$$

$$\frac{ab}{xy}$$

5)
$$\frac{3}{x} \div \frac{2}{y}$$

$$\frac{3}{x} \times \frac{y}{2}$$

$$= \frac{3y}{2x}$$

6)
$$\frac{1}{x+1} + \frac{1}{x+2}$$

$$\frac{1 \times (x+2) + 1 \times (x+1)}{(x+1)(x+2)}$$

7)
$$\frac{2}{x+1} + \frac{3}{x-2}$$

$$\frac{2(x-2) + 3(x+1)}{(x+1)(x-2)}$$

$$= \frac{2x-4 + 3x+3}{(x+1)(x-2)} = \frac{5x-1}{(x+1)(x-2)}$$

8)
$$\frac{x-1}{x+1} - \frac{x+1}{x+2}$$

$$= \frac{(x-1)(x+2) - (x+1)(x+1)}{(x+1)(x+2)}$$

$$= \frac{x^2+x-2 - (x^2+2x+1)}{(x+1)(x+2)}$$

9)
$$\frac{1}{x+1} \times \frac{1}{x+2}$$

$$= \frac{1}{(x+1)(x+2)}$$

$$= \frac{x^2+x-2 - x^2-2x-1}{(x+1)(x+2)}$$

$$= \frac{-x-3}{(x+1)(x+2)}$$

10)
$$\frac{1}{x+1} \div \frac{1}{x+2}$$

$$\frac{1}{x+1} \times \frac{x+2}{1} = \frac{x+2}{x+1}$$