ALGEBRA - PROOF

Algebraic Proof - Two types of questions

Type 1 - Come to a conclusion by considering several examples

1) The number a is prime and b is even.

Is a + b even, odd or either

Is a x b even, odd or either

You must write down several examples to cover all the possibilities.
Remember for prime numbers to consider 2 the only even prime.
Remember to answer the question.

a prime 2, 3, 5, 7, 11 . . . . .

b even 2, 4, 6, 8, 10 . . . . .

a+b 2 + 4 = 6 even
3 + 6 = 9 odd

could be either

axb 2 x 4 = 8 even
3 x 6 = 18 even

even x even = even
odd x even = even

Always even

2) 'q' is an odd whole number and 'p' is an even whole number. Answer the following questions.

p 2, 4, 6, 8 . . . . q 1, 3, 5, 7, ...

a) Is pq odd even could be either

4 x 3 = 12

even x odd is always even.

b) Is 3(p + q) odd even could be either

3 x (2 + 3)
3 x 5 = 15

even + odd = odd
3 x odd = odd

c) Is p ÷ q an integer not an integer could be either

$\frac{12}{3} = 4$ integer

$\frac{2}{5} = 0.4$ not an integer.

So could be either.

Type 2 - Using algebra construct a proof

Helpful results - if n is an integer
2n is always even
Consecutive even numbers are 2n and 2n + 2
2n + 1 is always odd
Consecutive odd numbers are 2n + 1 and 2n + 3
4n is a muliple of 4
3(n + 2) is a multiple of 3 etc..

1) n is an integer (whole number)

Is $(n + 1)^2 - (n - 1)^2$ even or odd?

$(n+1)^2 = n^2 + 2n + 1$

$(n-1)^2 = n^2 - 2n + 1$

$(n+1)^2 - (n-1)^2$

$= n^2 + 2n + 1 - (n^2 - 2n + 1)$

$= n^2 + 2n + 1 - n^2 + 2n - 1$

$= 4n$

$4n$ is always even. It is a multiple of 4

2) Prove that the difference between the squares of two consecutive odd numbers is always a muliple of 8.

$(2n+3)^2 - (2n+1)^2$

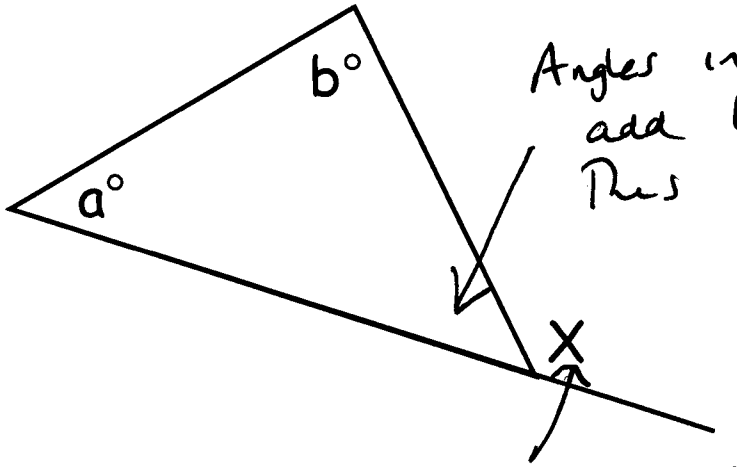
$= (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$

$= 4n^2 + 12n + 9 - 4n^2 - 4n - 1$

$= 8n + 8$

$= 8(n+1)$ a multiple of 8

3) Prove that angle X is equal to a + b



Angles in a triangle add to 180
Thus angle is
180 - a - b

angles on a straight line add to 180

$X = 180 - (180 - a - b)$

$= 180 - 180 + a + b$

$X = a + b.$